



S.S. PAPADOPULOS & ASSOCIATES, INC.
ENVIRONMENTAL AND WATER RESOURCE CONSULTANTS

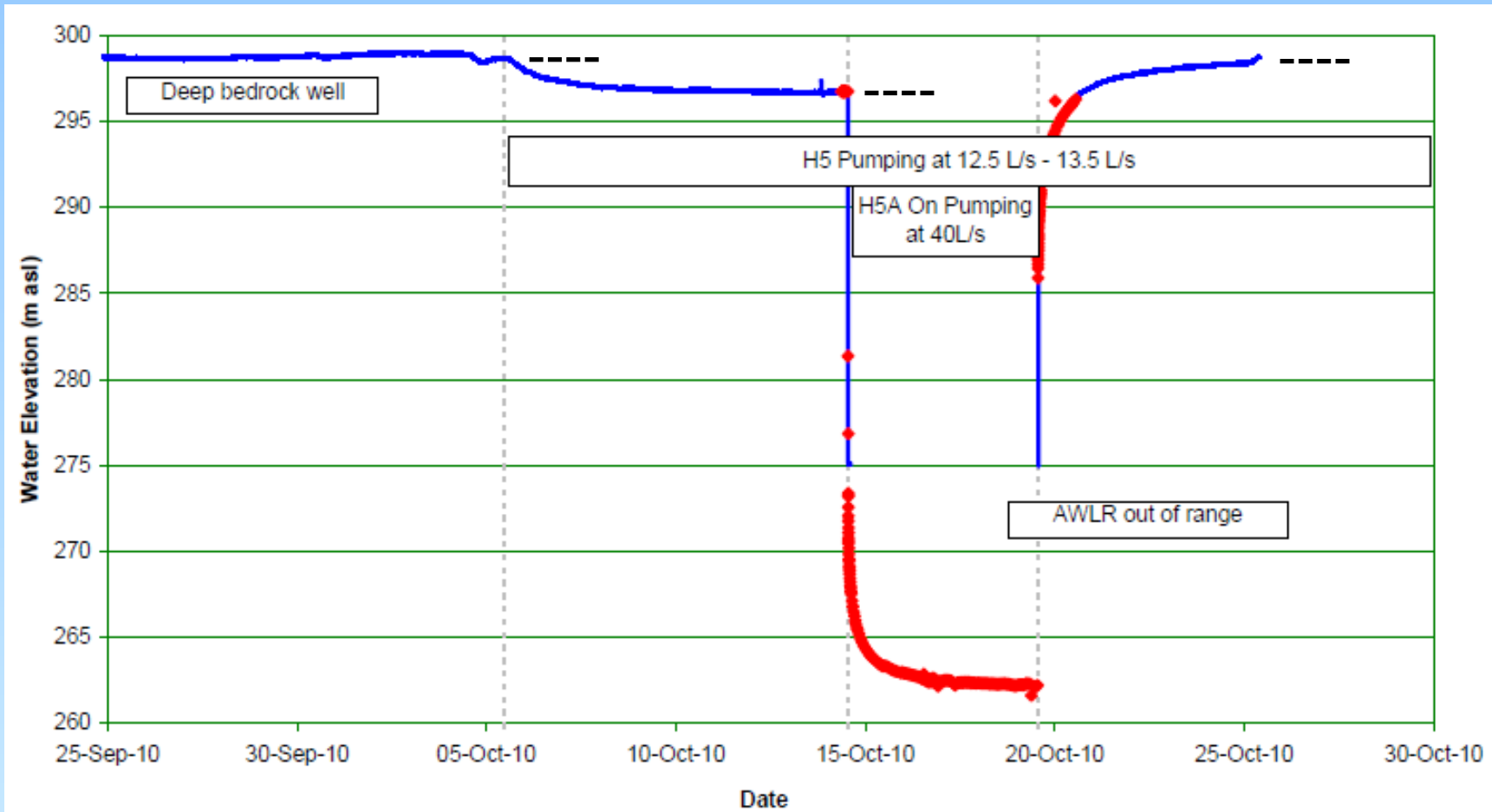
The Significance and Interpretation of Recovery Data

Christopher J. Neville
S.S. Papadopoulos & Associates, Inc.

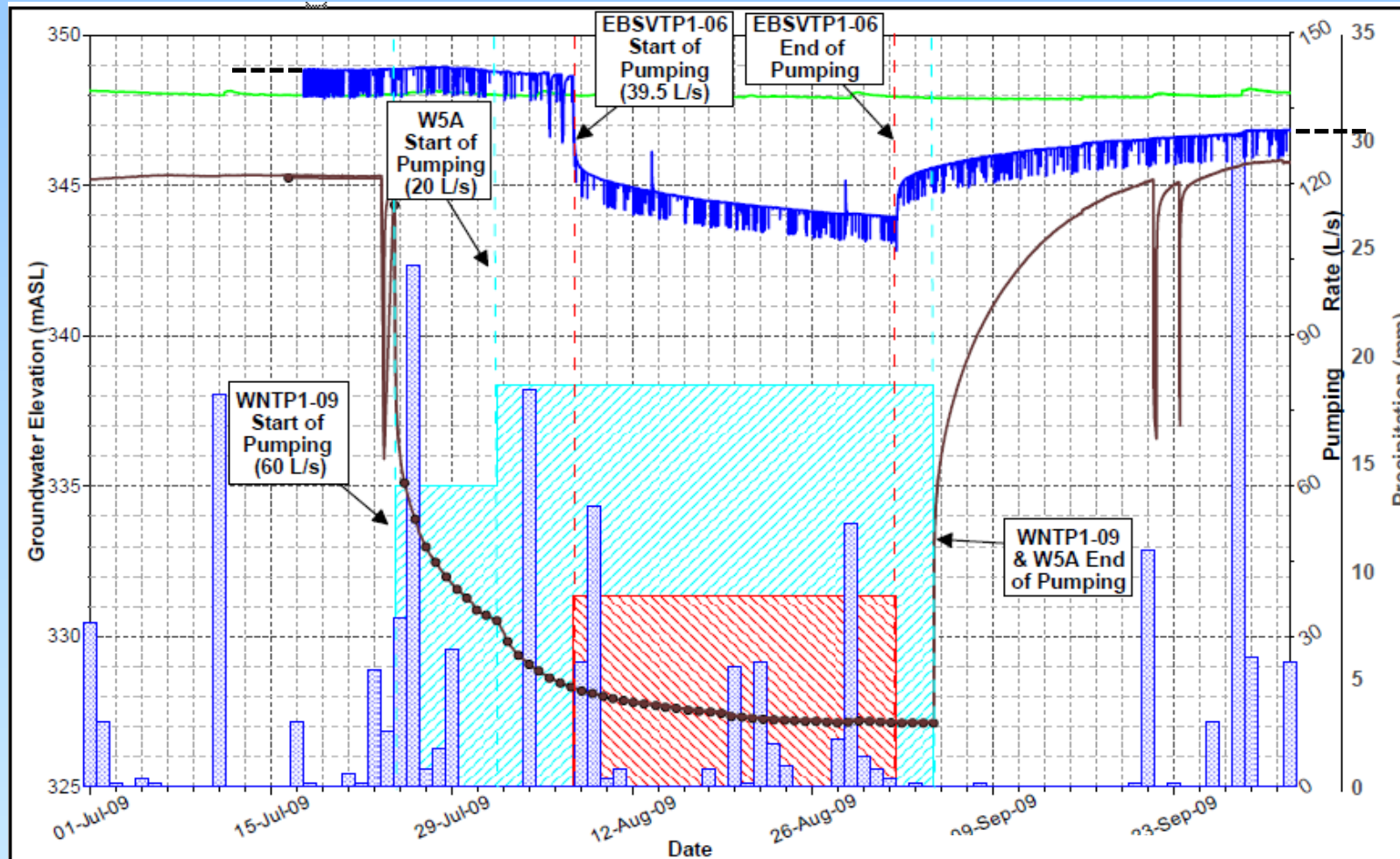
Importance of recovery data

1. Recovery data serve to identify background trends in water levels during a pumping test.
2. Sometimes all we get are recovery data.
3. Recovery data are “smoother”.
4. Recovery data may provide a useful check on the interpretations of drawdowns.
5. Recovery data can provide insights that the drawdown data may not.
6. Recovery data can be used to extend the effective duration of pumping.

Use of recovery data to check on background trends during pumping



Use of recovery data to check on background trends during pumping (2)

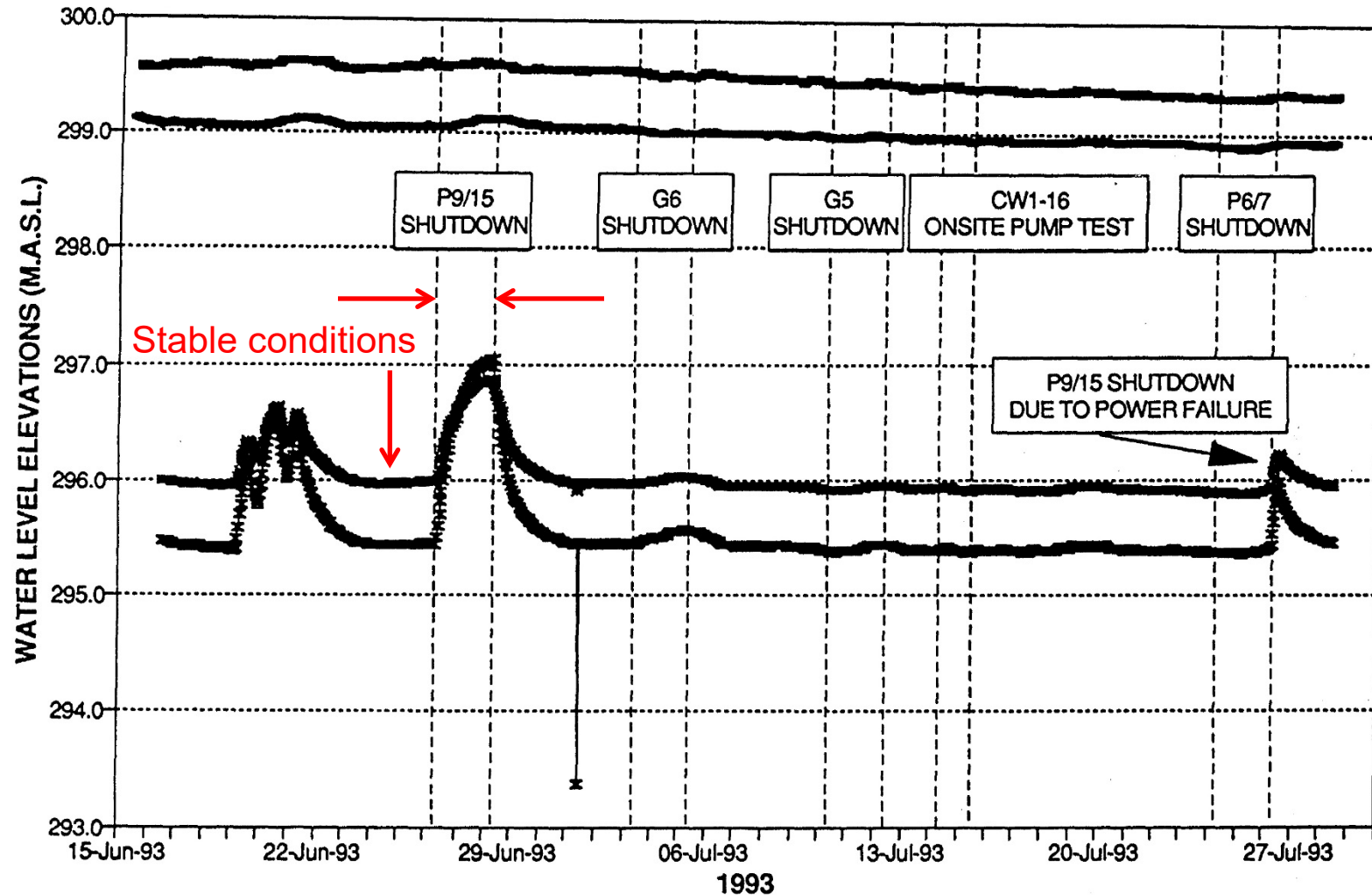


Making do

Sometimes it is only feasible to collect recovery data. This is particularly true for the case of operating municipal wellfields.

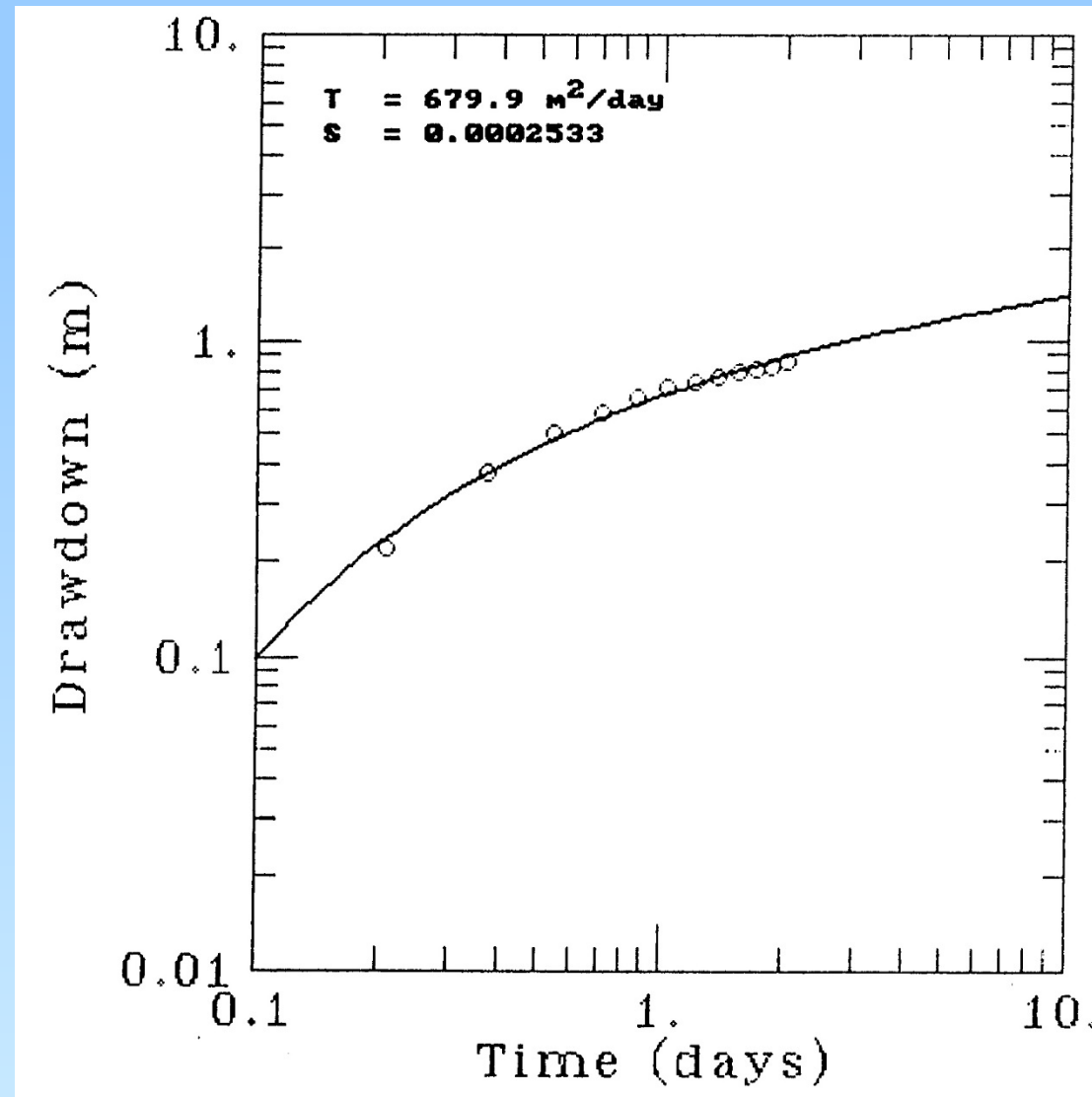
With a bit of planning and communication with municipal operators, it may be possible to turn a scheduled shutdown into an effective test.

City of Cambridge, Ontario: Well shutdown test



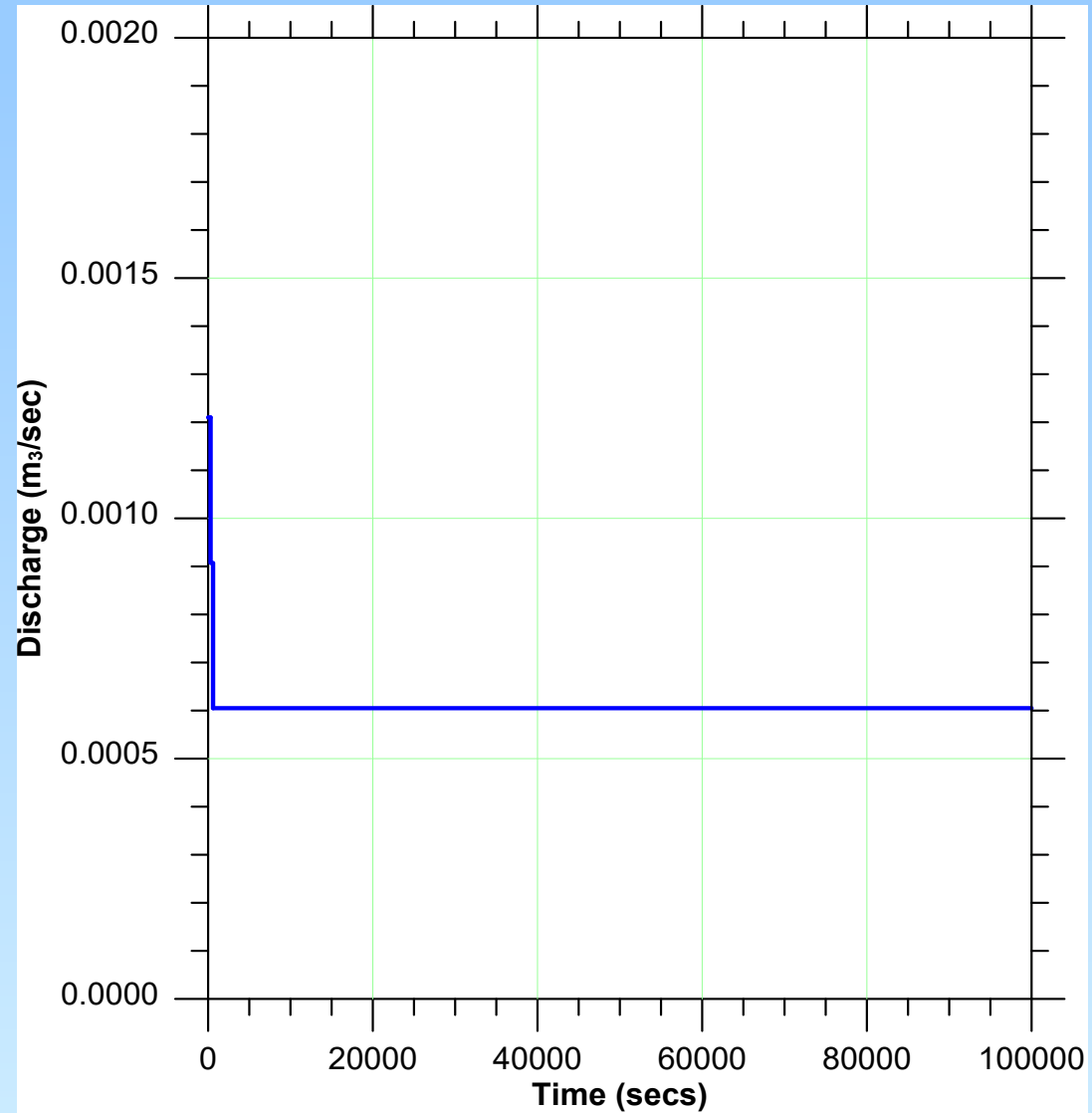
In this case, the shutdown test is in fact a pumping test run in reverse.

Interpretation of well shutdown test

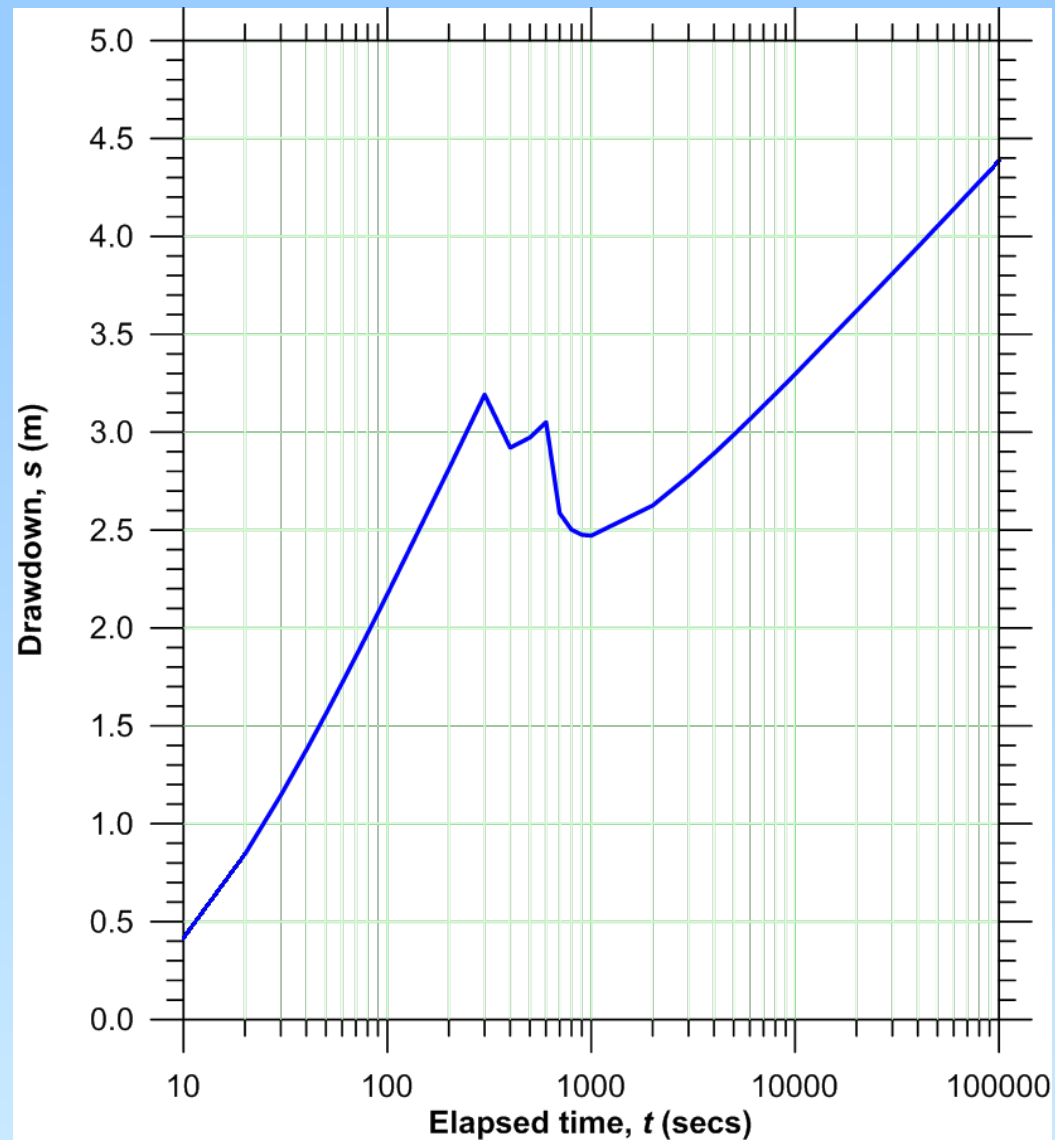


Recovery data are generally smoother than drawdown data

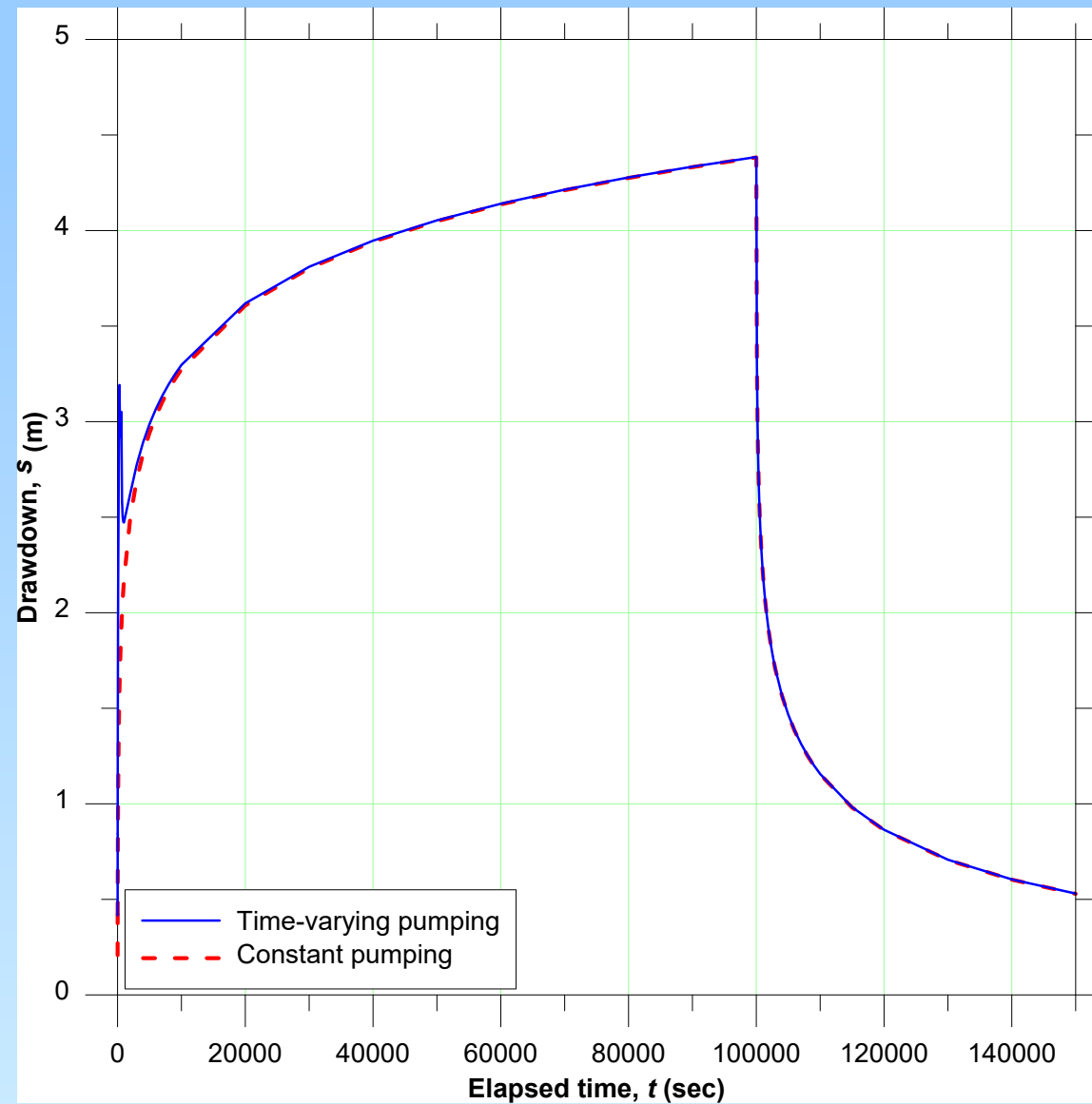
Smoothing effect:
Example #1



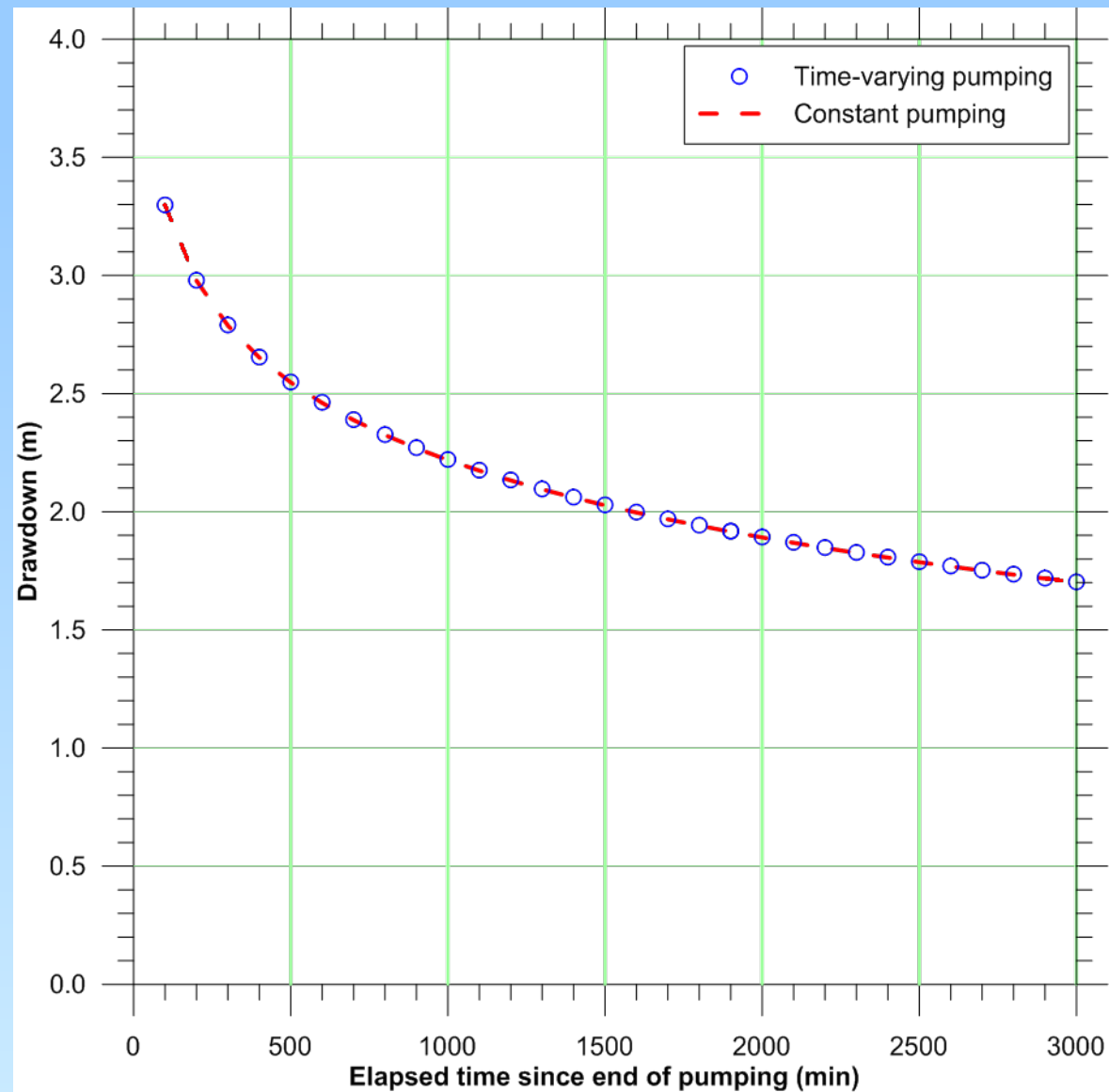
Drawdown record



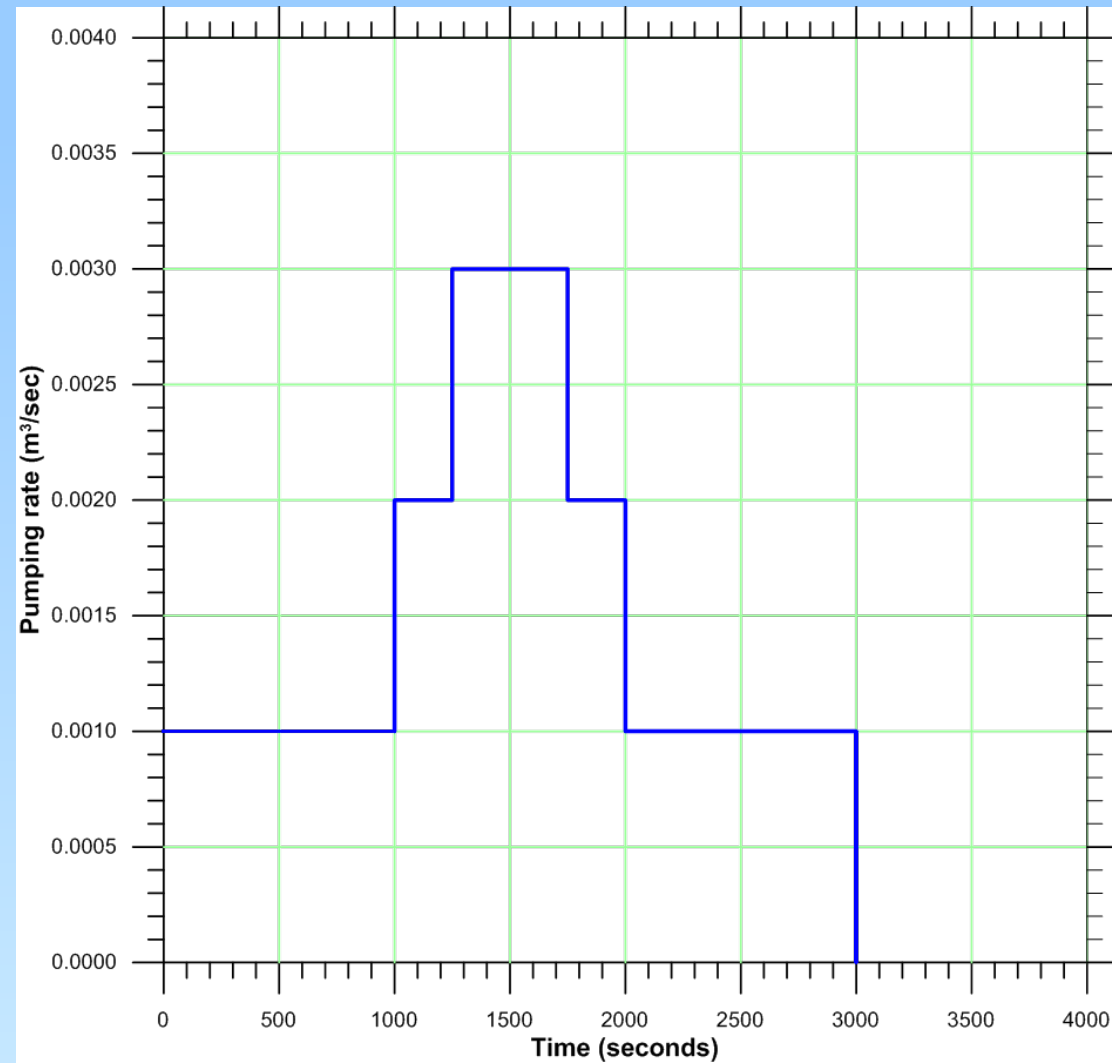
Complete time-drawdown record



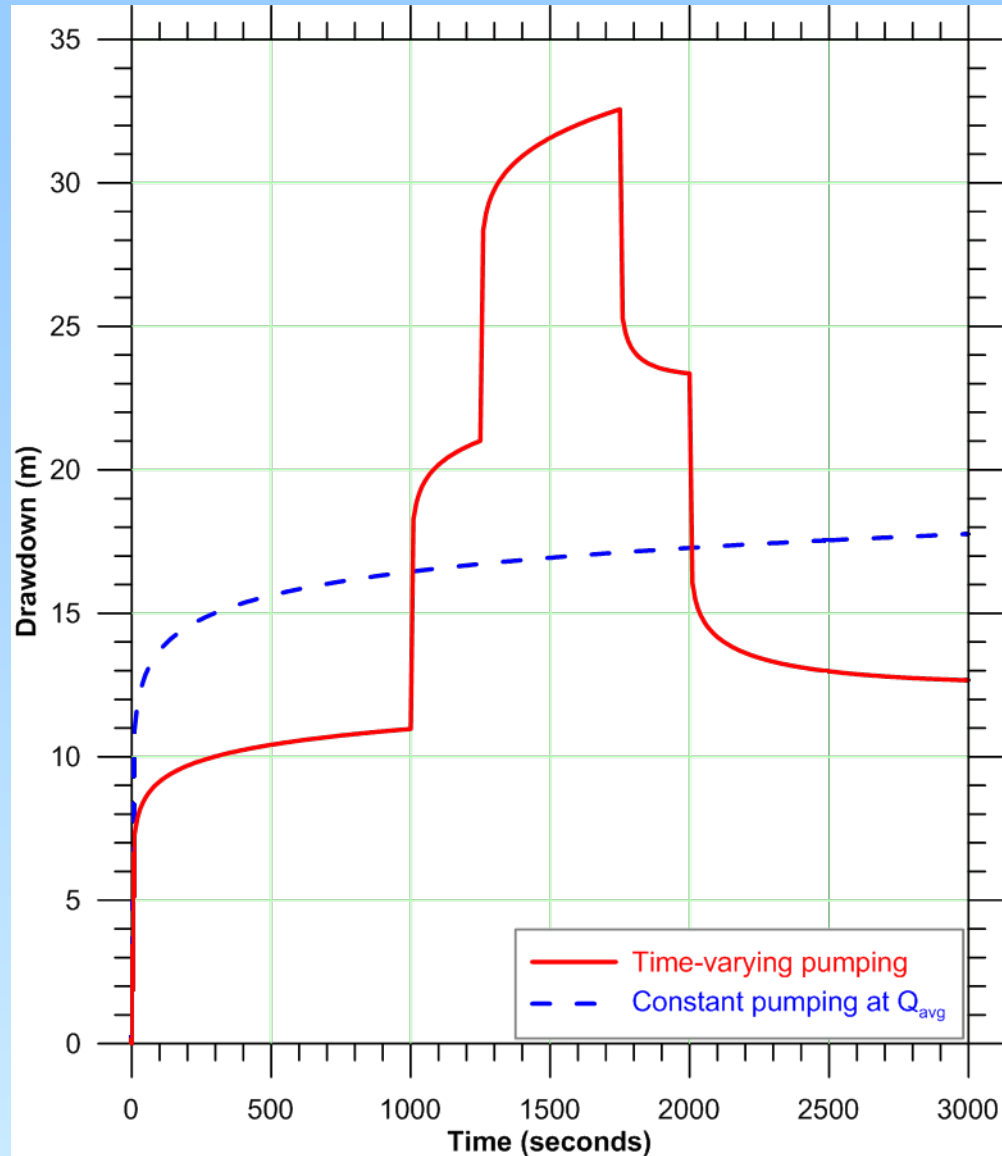
Plot of recovery



Smoothing effect: Example #2

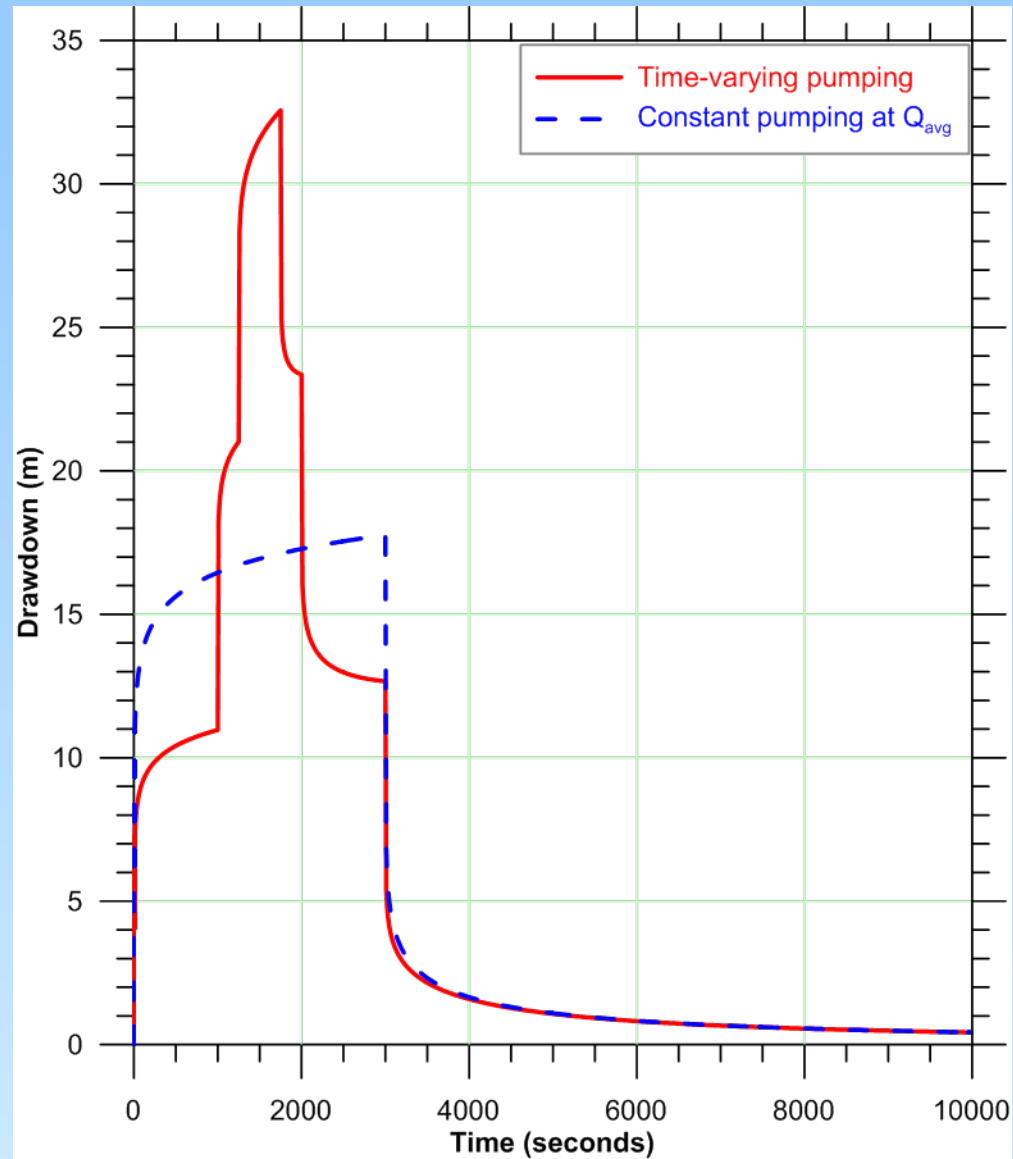


Drawdown record

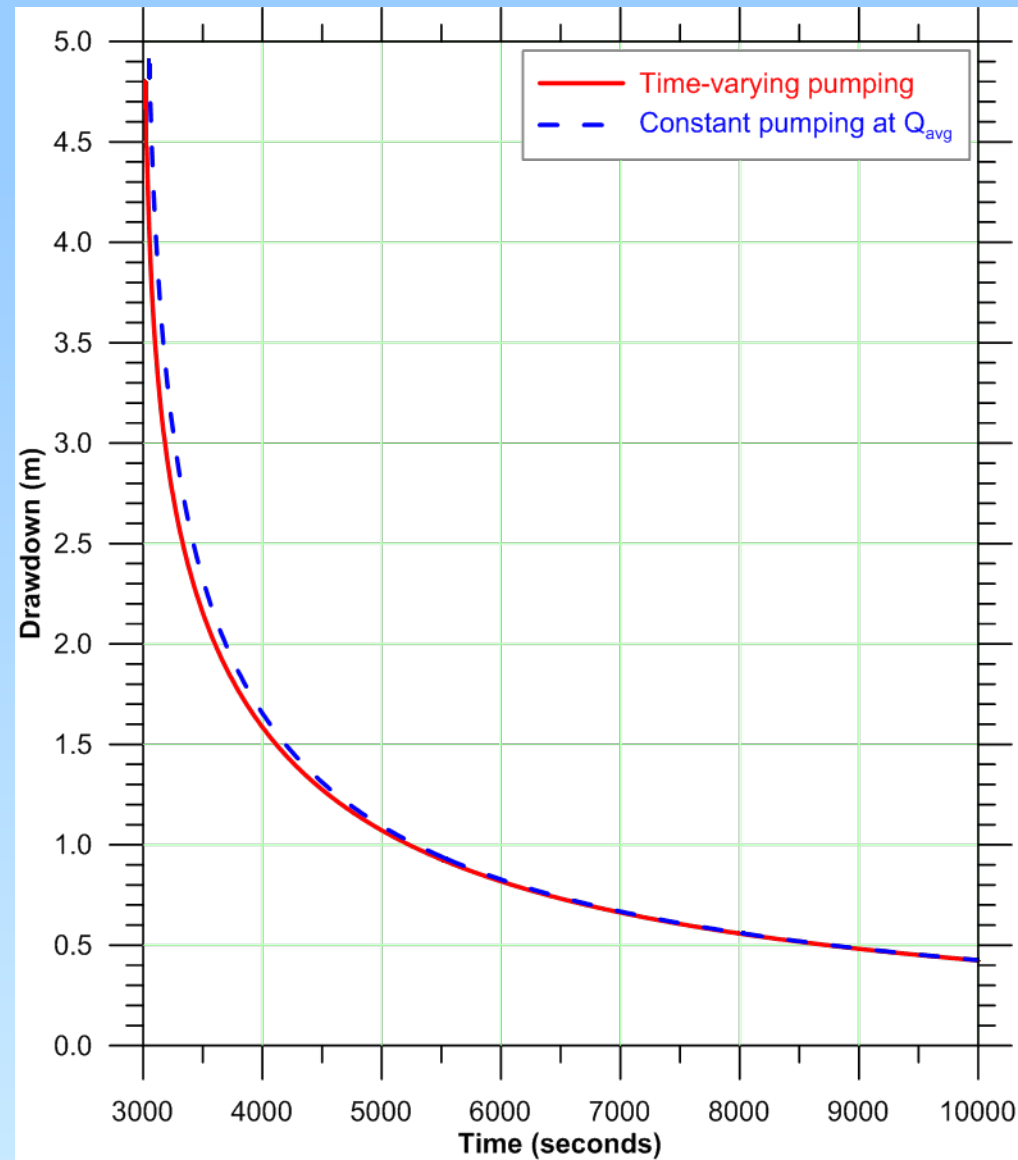


Q_{avg} is defined as that constant rate which extracts the same volume of water by the end of pumping.

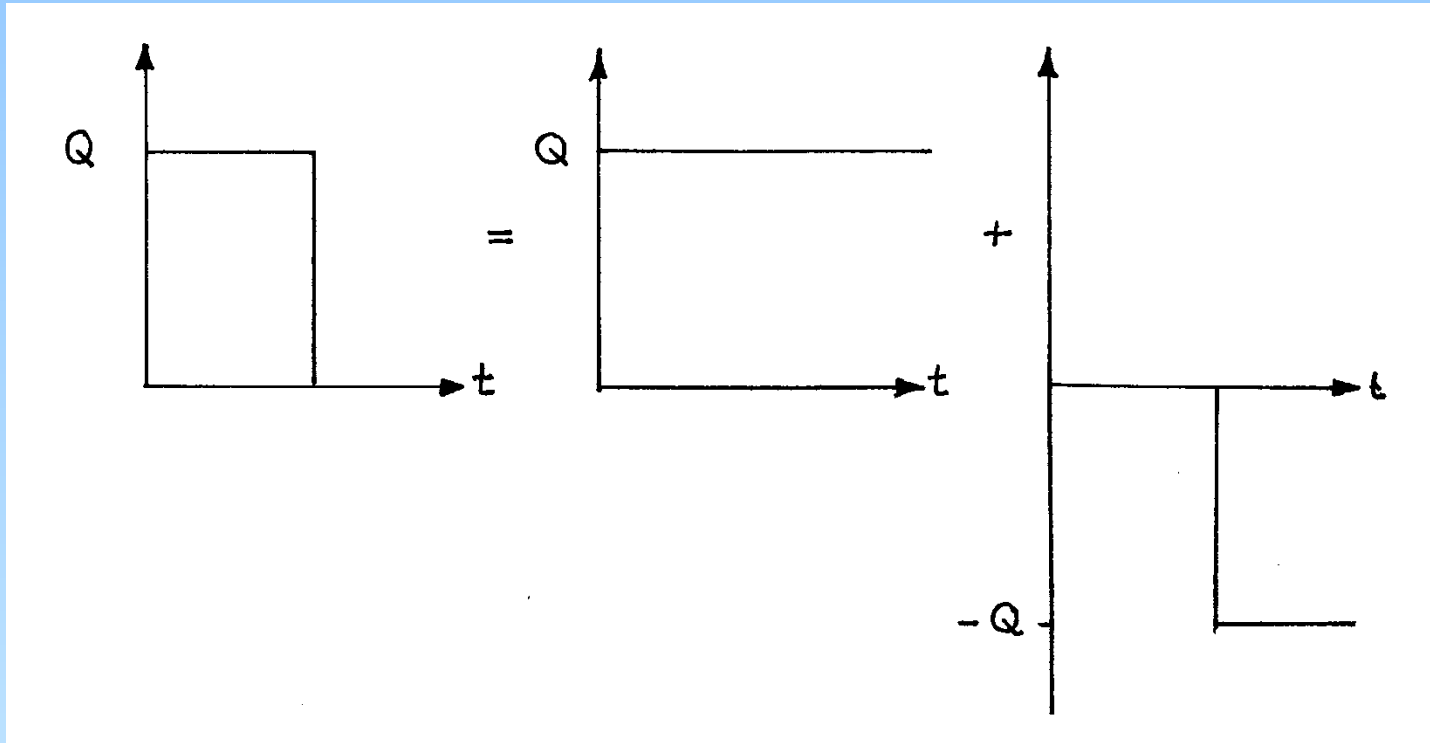
Complete time-drawdown record



Plot of recovery



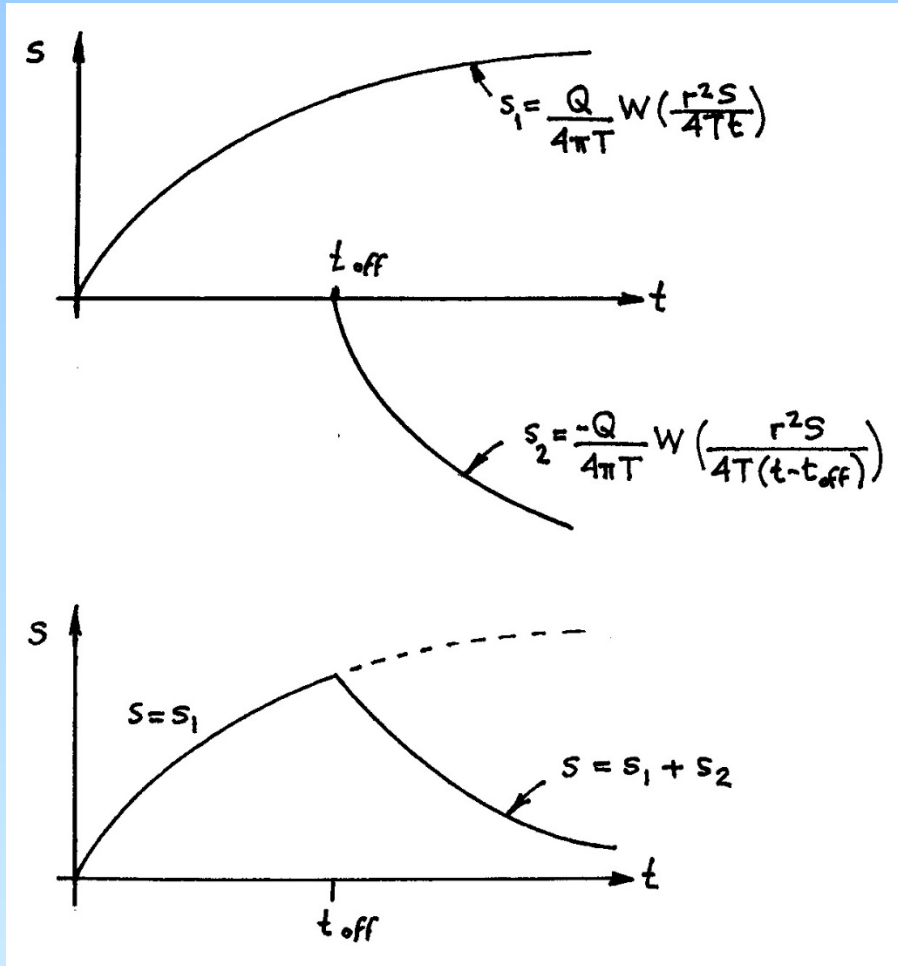
Interpretation of recovery data: Principle of Superposition



Superposition is valid for any linear aquifer model.

Superposition: Theis model

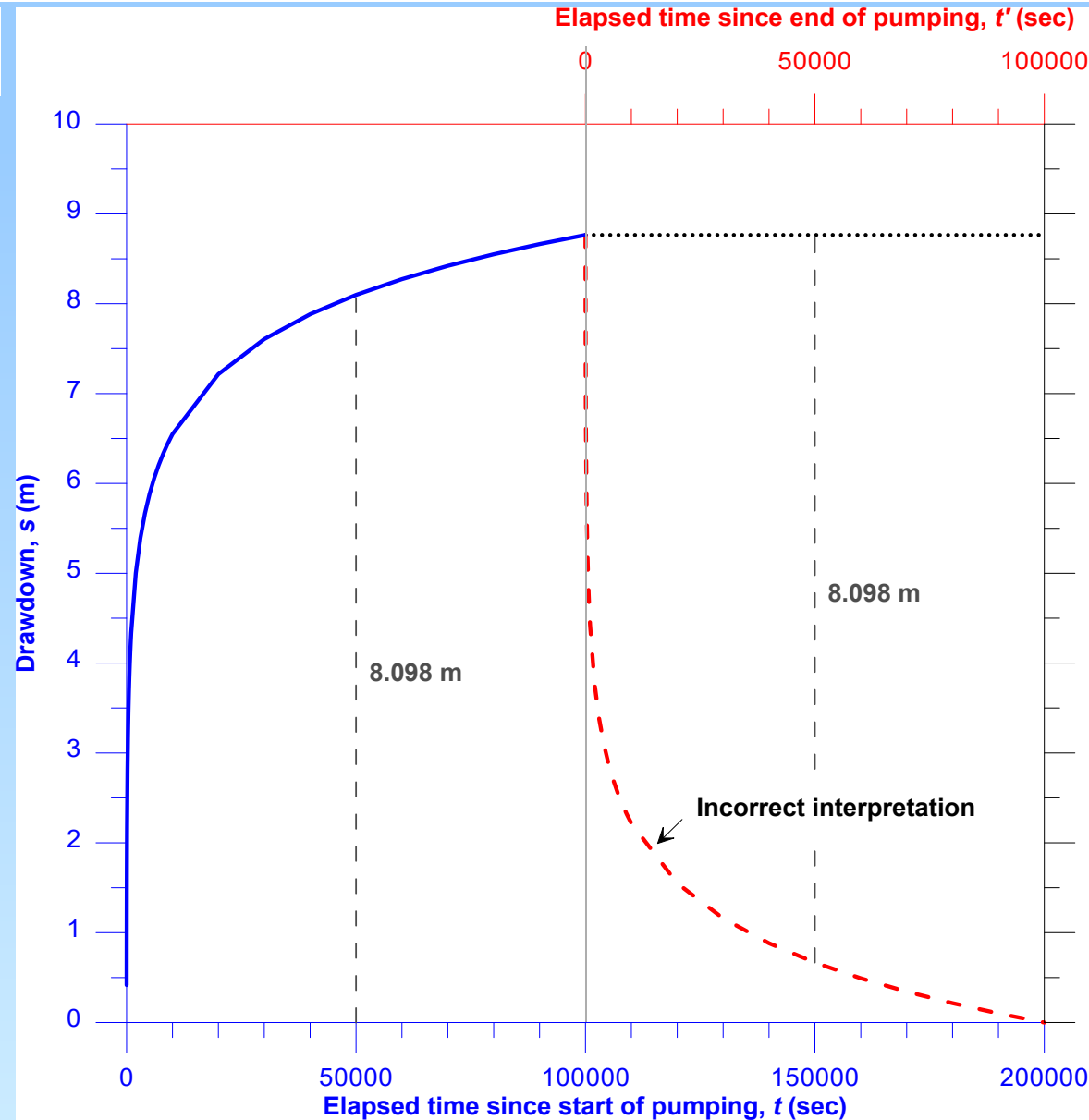
$$s(r, t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4T(t - t_{off})}\right)$$



A common misconception:

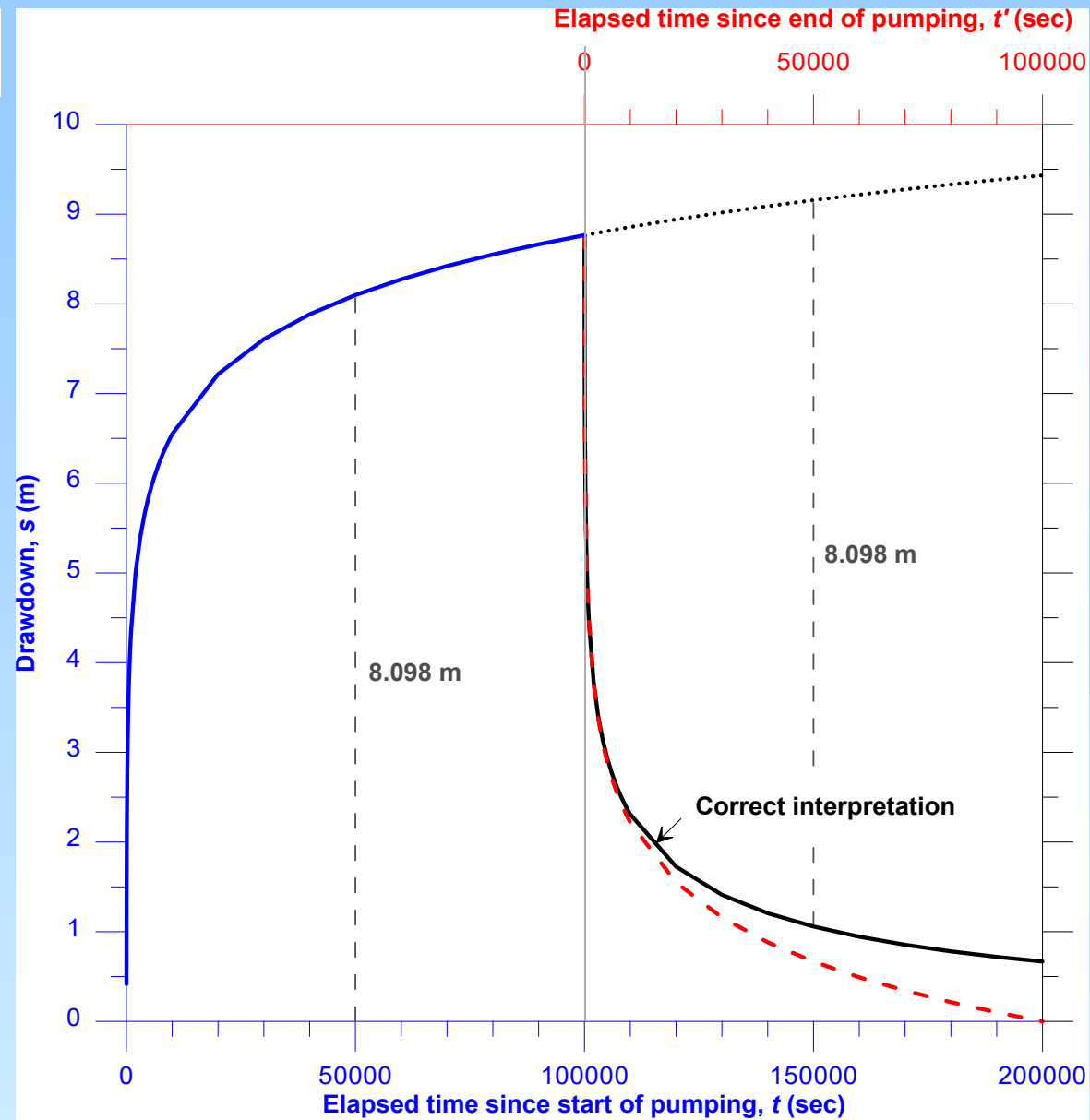
Transient recovery is usually not a mirror image of drawdown

Incorrect



Correct

Recovery = Drawdown only if water levels have stabilized by the end of pumping.



Semilog recovery analysis

Application to recovery following
pumping at a constant rate

Replacing $W(u)$ with its approximation

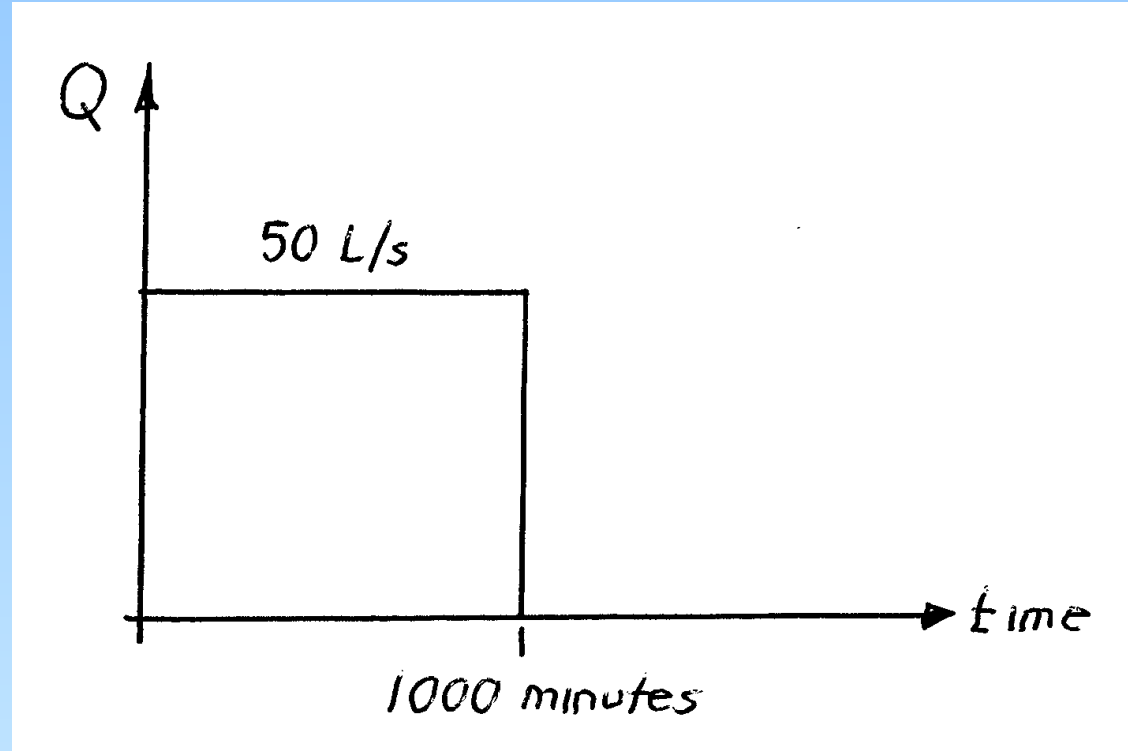
$$s(r, t) = \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S}{4Tt} \right\} \right] - \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S}{4T(t - t_{off})} \right\} \right]$$

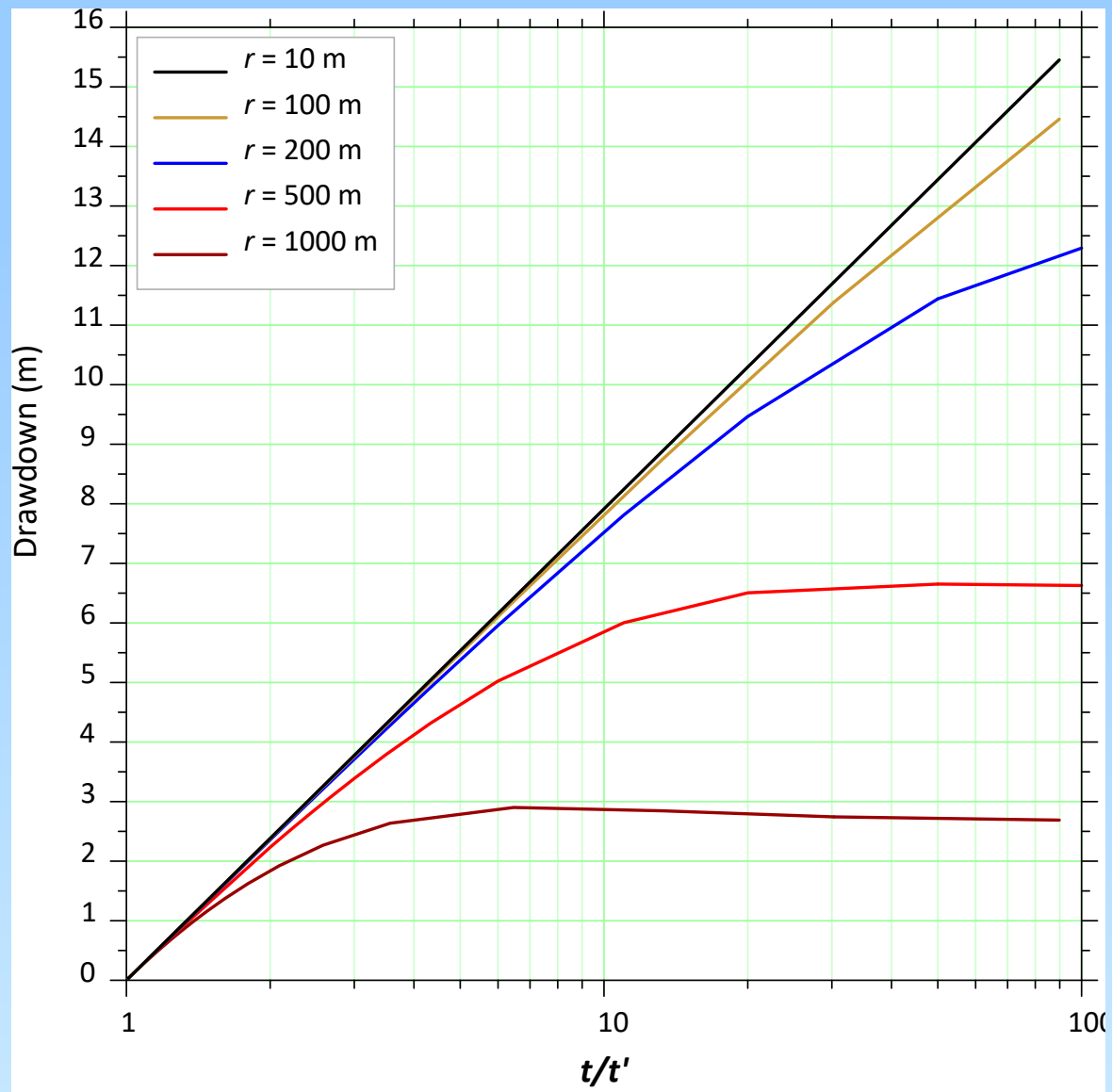
$$\begin{aligned} s(r, t) &= \frac{Q}{4\pi T} \ln \left\{ \frac{\left(\frac{r^2 S}{4T(t - t_{off})} \right)}{\left(\frac{r^2 S}{4Tt} \right)} \right\} \\ &= 2.303 \frac{Q}{4\pi T} \log_{10} \left\{ \frac{t}{t - t_{off}} \right\} \end{aligned}$$

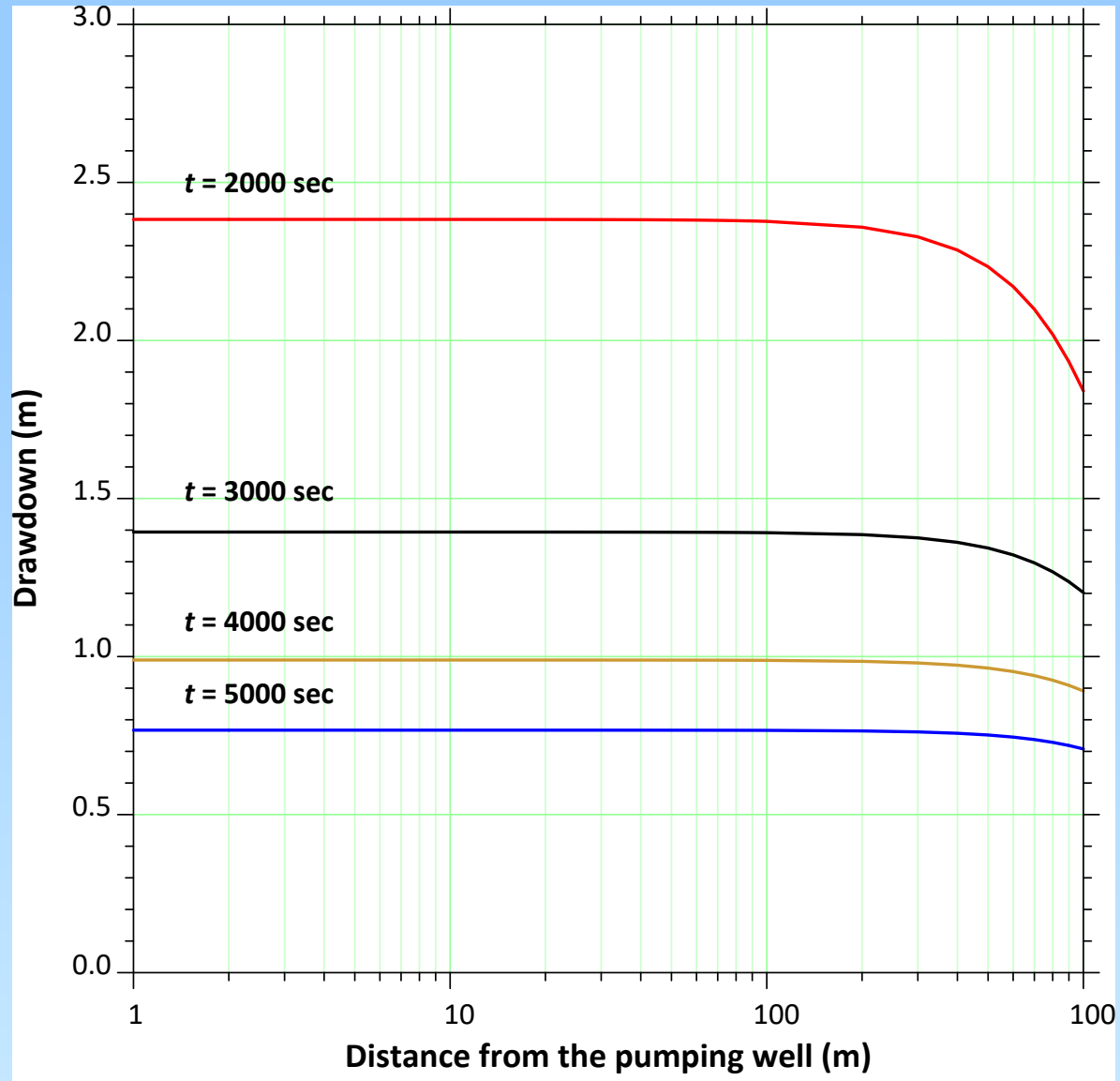
Where are r and S ?

Example analysis

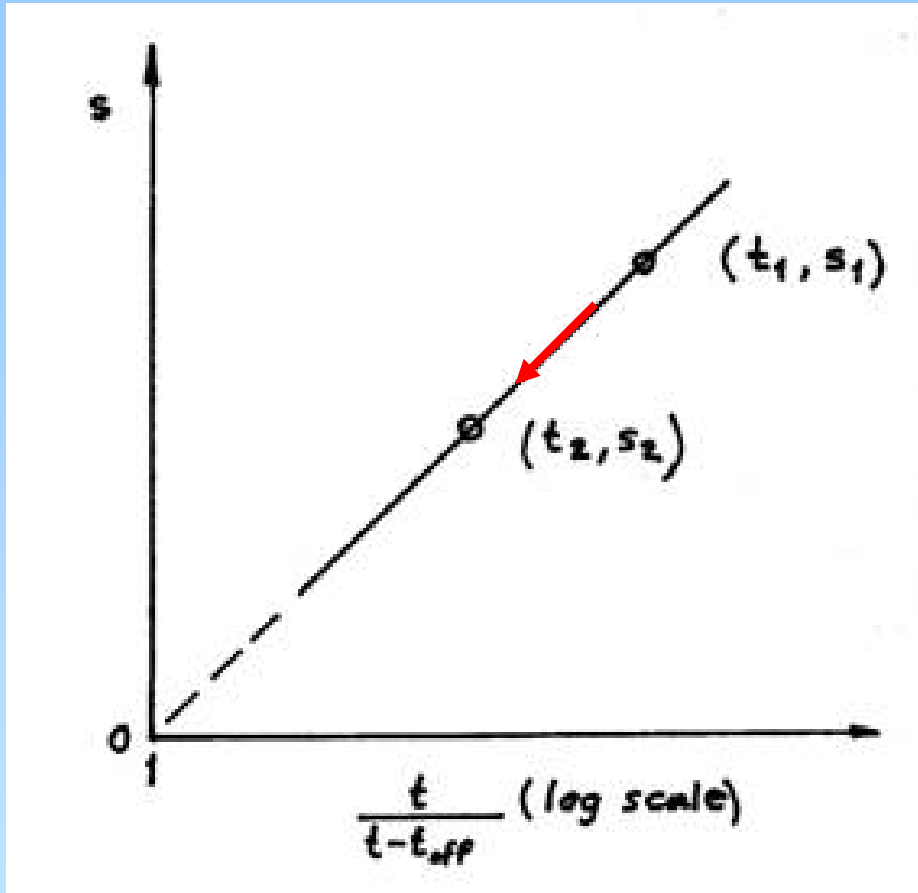
$$T = 100 \text{ m}^2/\text{d}$$
$$S = 1.0 \times 10^{-4}$$







Straight-line recovery analysis



$$s(r, t_1) = 2.303 \frac{Q}{4\pi T} \log_{10} \left\{ \frac{t_1}{t_1 - t_{off}} \right\}$$

$$s(r, t_2) = 2.303 \frac{Q}{4\pi T} \log_{10} \left\{ \frac{t_2}{t_2 - t_{off}} \right\}$$

$$s(r, t_1) - s(r, t_2) =$$

$$2.303 \frac{Q}{4\pi T} \log_{10} \left\{ \frac{\left\{ \frac{t_1}{t_1 - t_{off}} \right\}}{\left\{ \frac{t_2}{t_2 - t_{off}} \right\}} \right\}$$

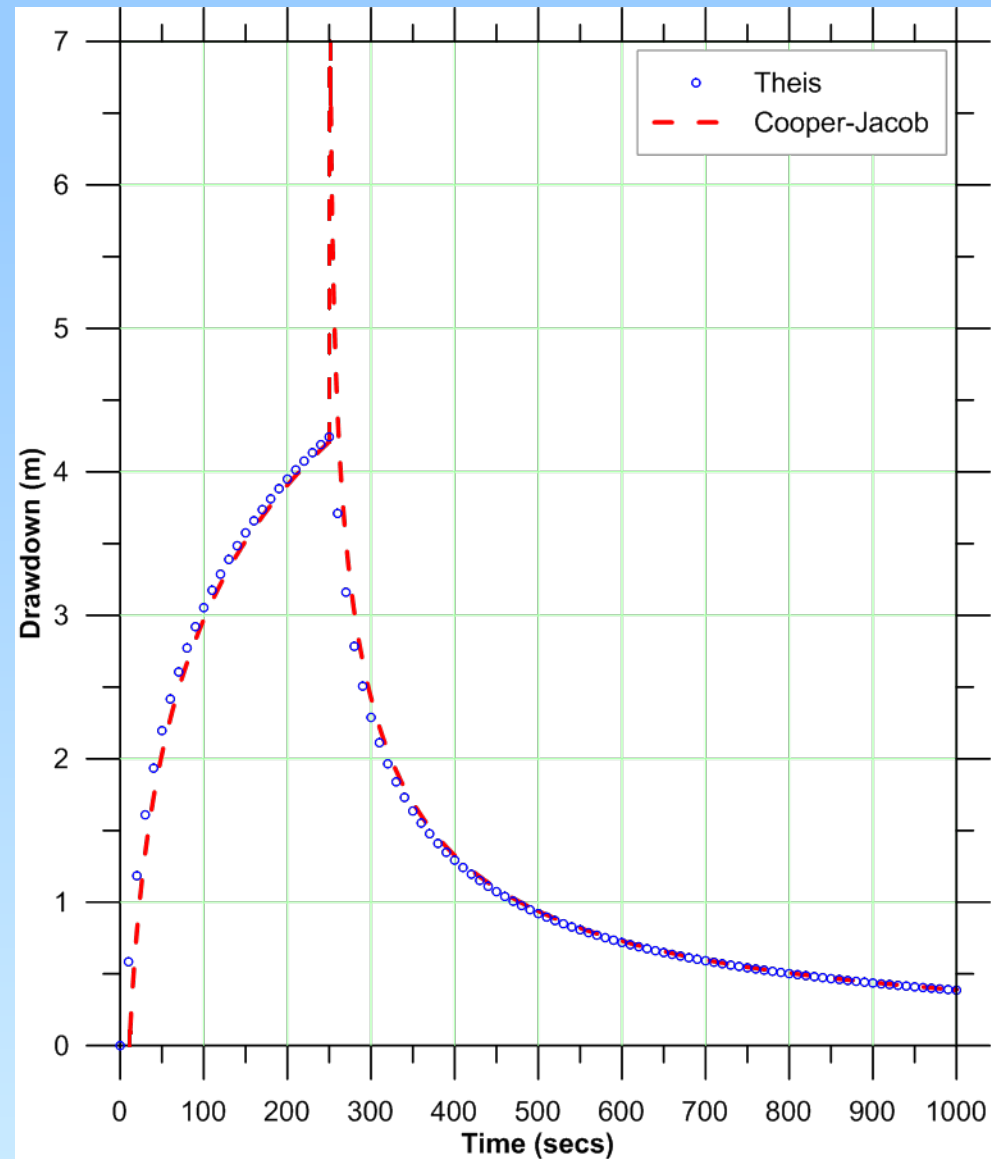
$$T = 2.303 \frac{Q}{4\pi} \frac{1}{SLOPE}$$

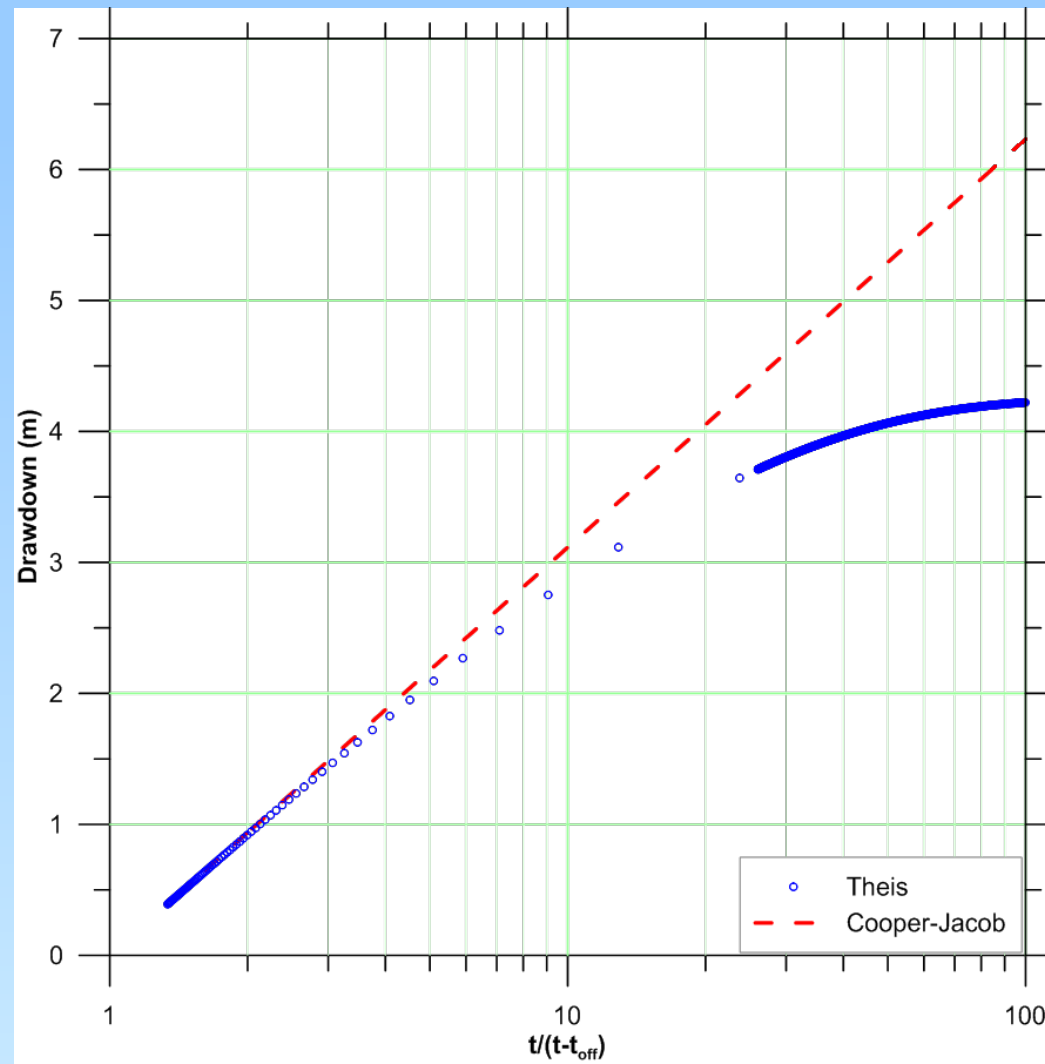
Example

$$T = 1.0 \times 10^{-4} \text{ m}^2/\text{s}$$

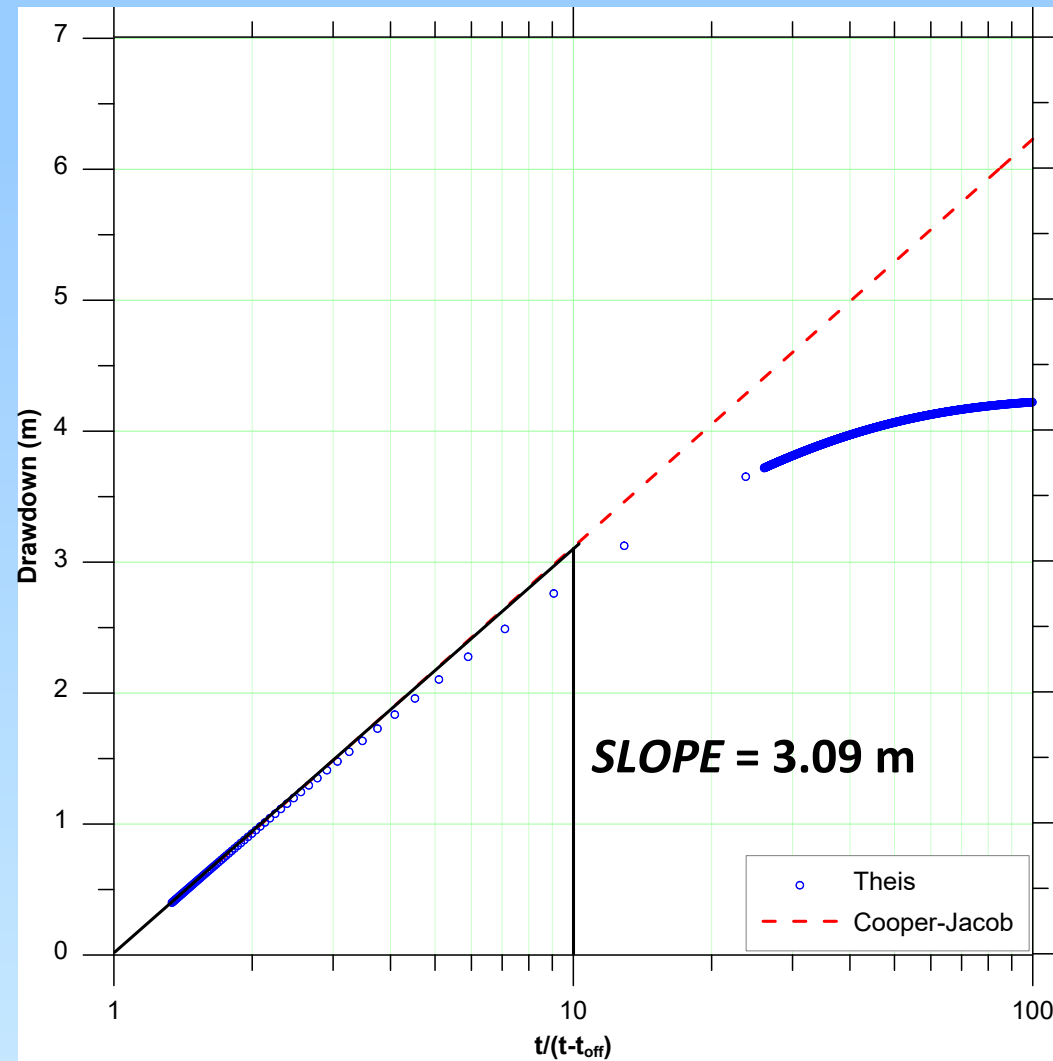
$$S = 1.0 \times 10^{-4}$$

$$Q = 1.7 \times 10^{-3} \text{ m}^3/\text{s}$$

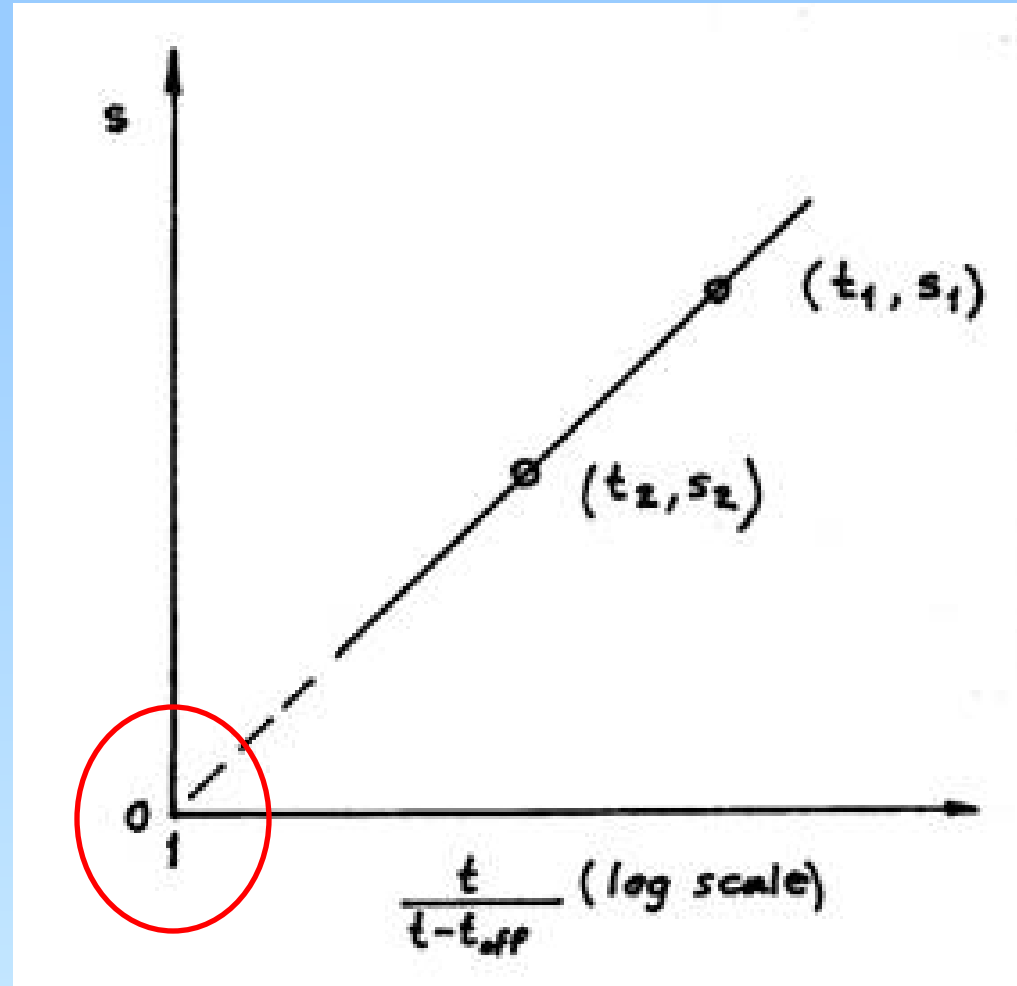




Estimation of transmissivity



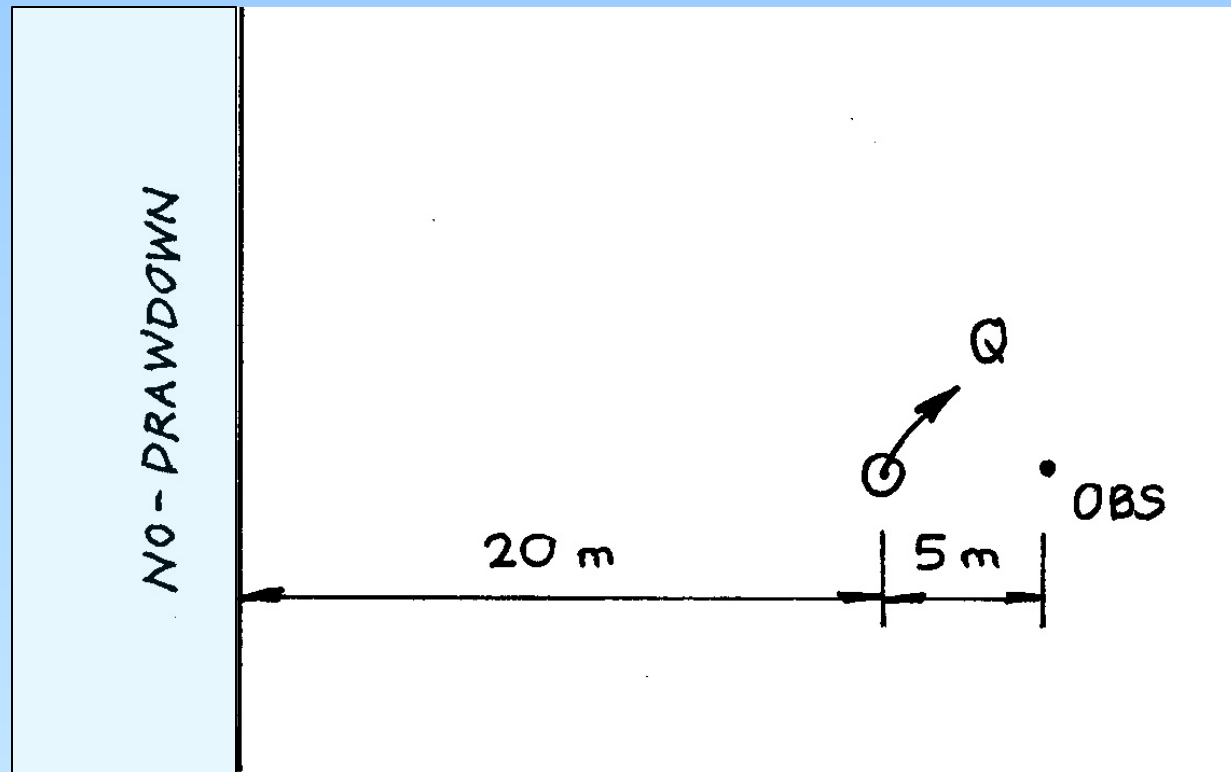
Asymptote for the Cooper-Jacob plot

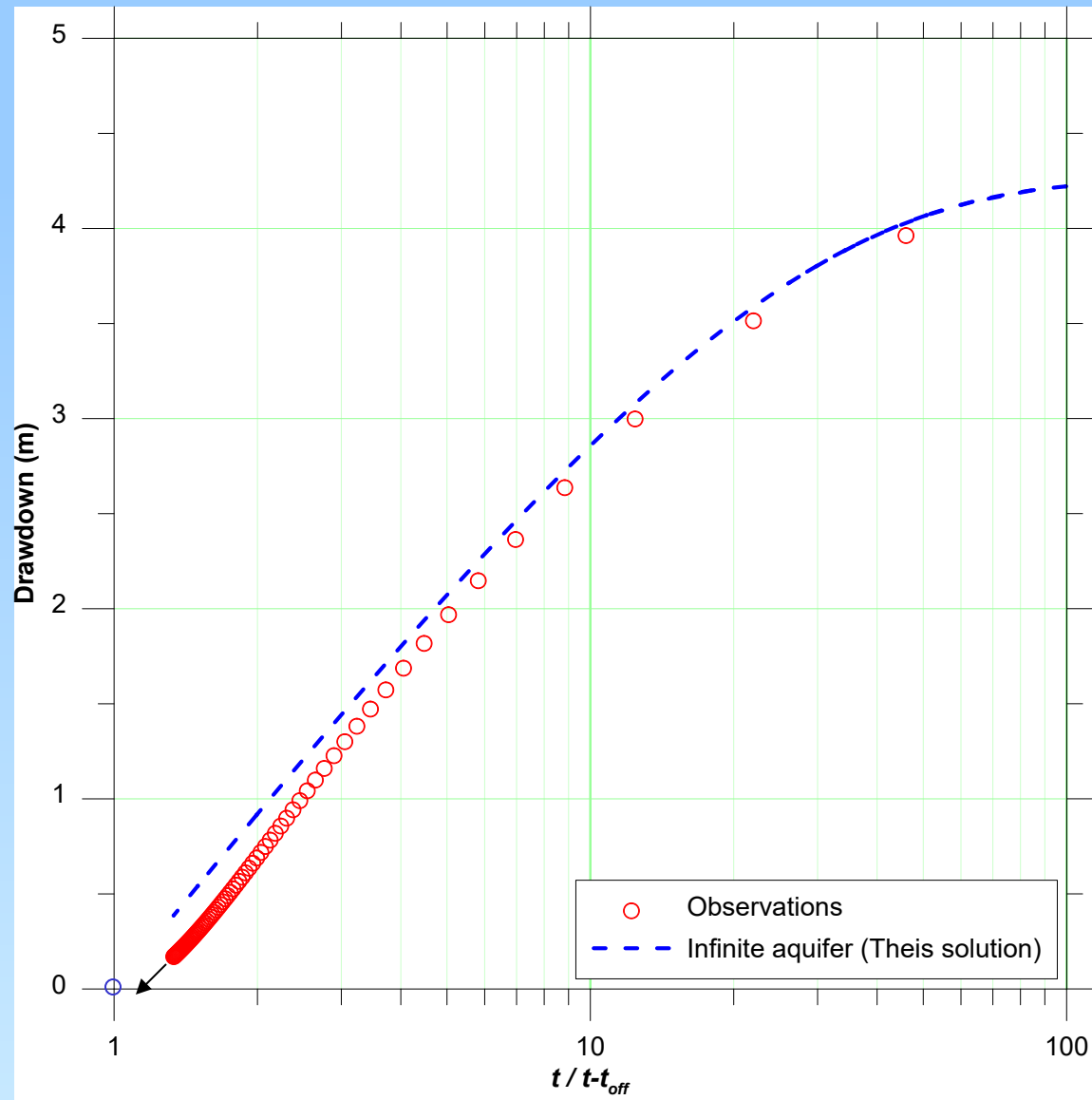


$$s(r,t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

Example:

Pumping near a linear recharge boundary



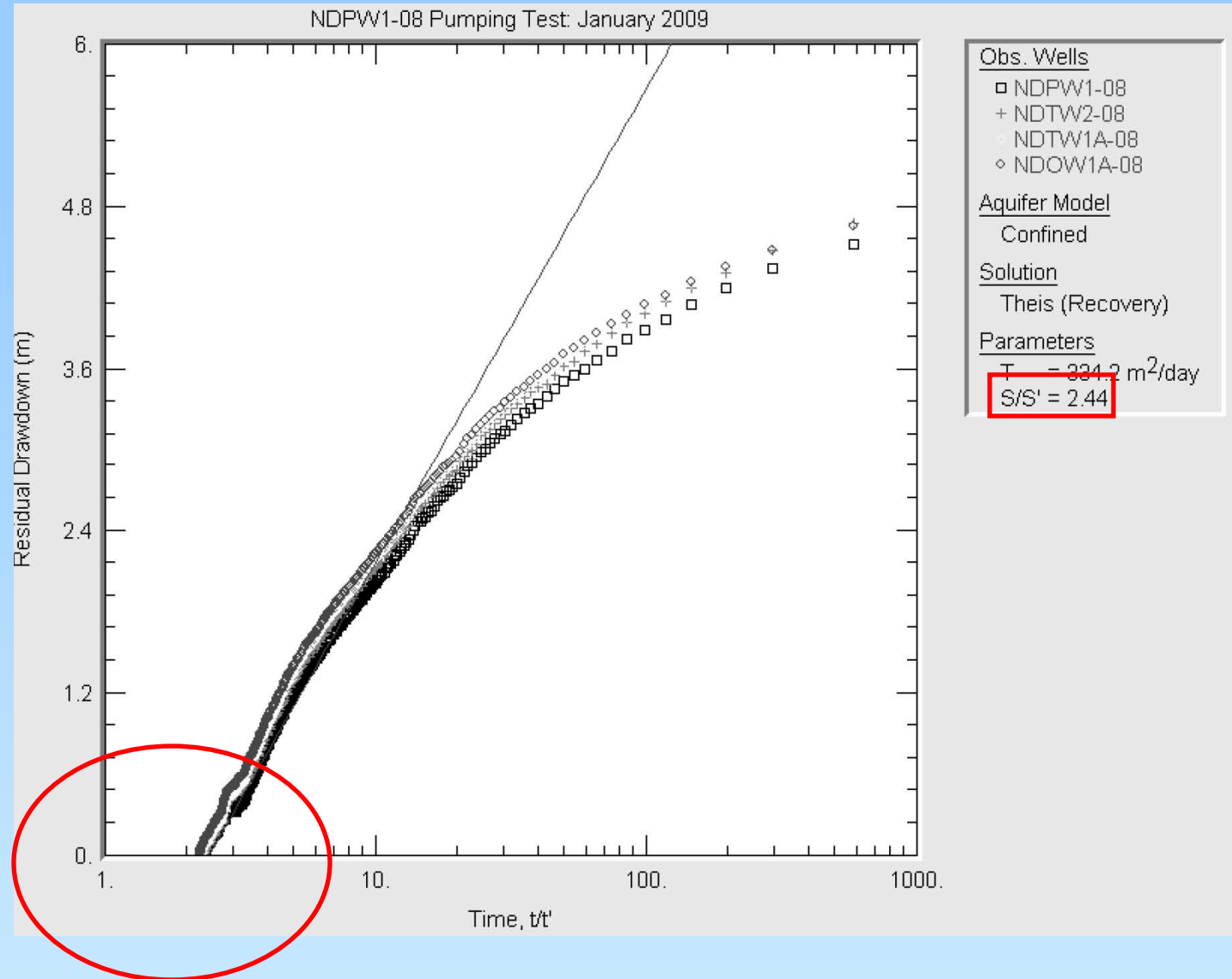


Something wrong but nevertheless useful

$$s(r, t) = \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S}{4Tt} \right\} \right] - \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S'}{4T(t - t_{off})} \right\} \right]$$

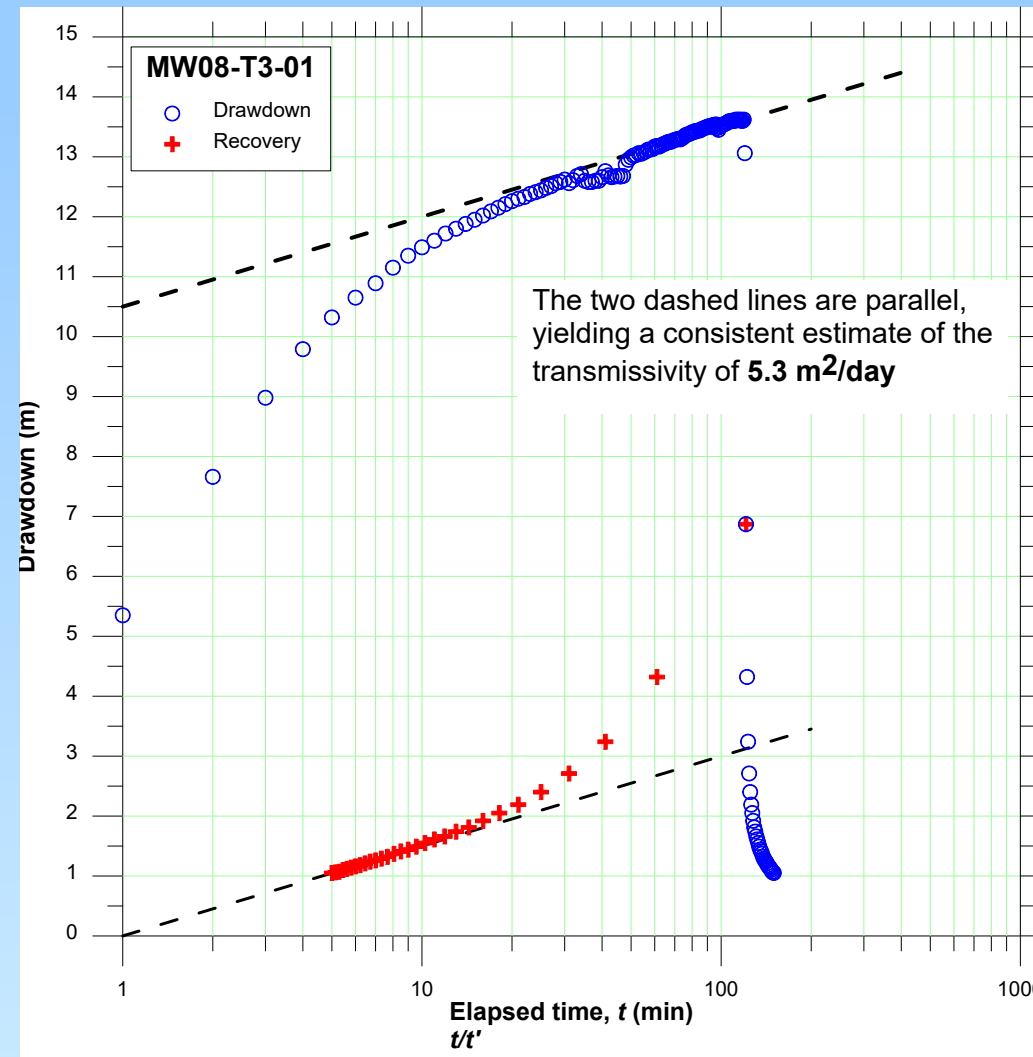
$$s(r, t) = \frac{Q}{4\pi T} \ln \left\{ \frac{\left(\frac{r^2 S'}{4T(t - t_{off})} \right)}{\left(\frac{r^2 S}{4Tt} \right)} \right\}$$
$$= \frac{Q}{4\pi T} 2.303 \log_{10} \left\{ \frac{t}{t - t_{off}} \frac{1}{S/S'} \right\}$$

Example: Cambridge, Ontario



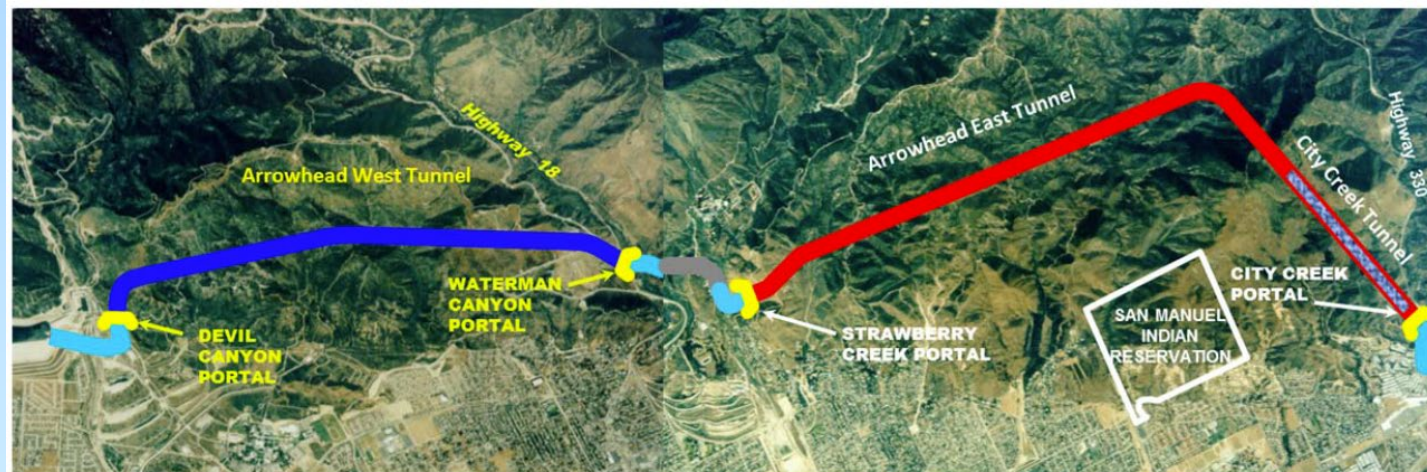
Applications of recovery analysis

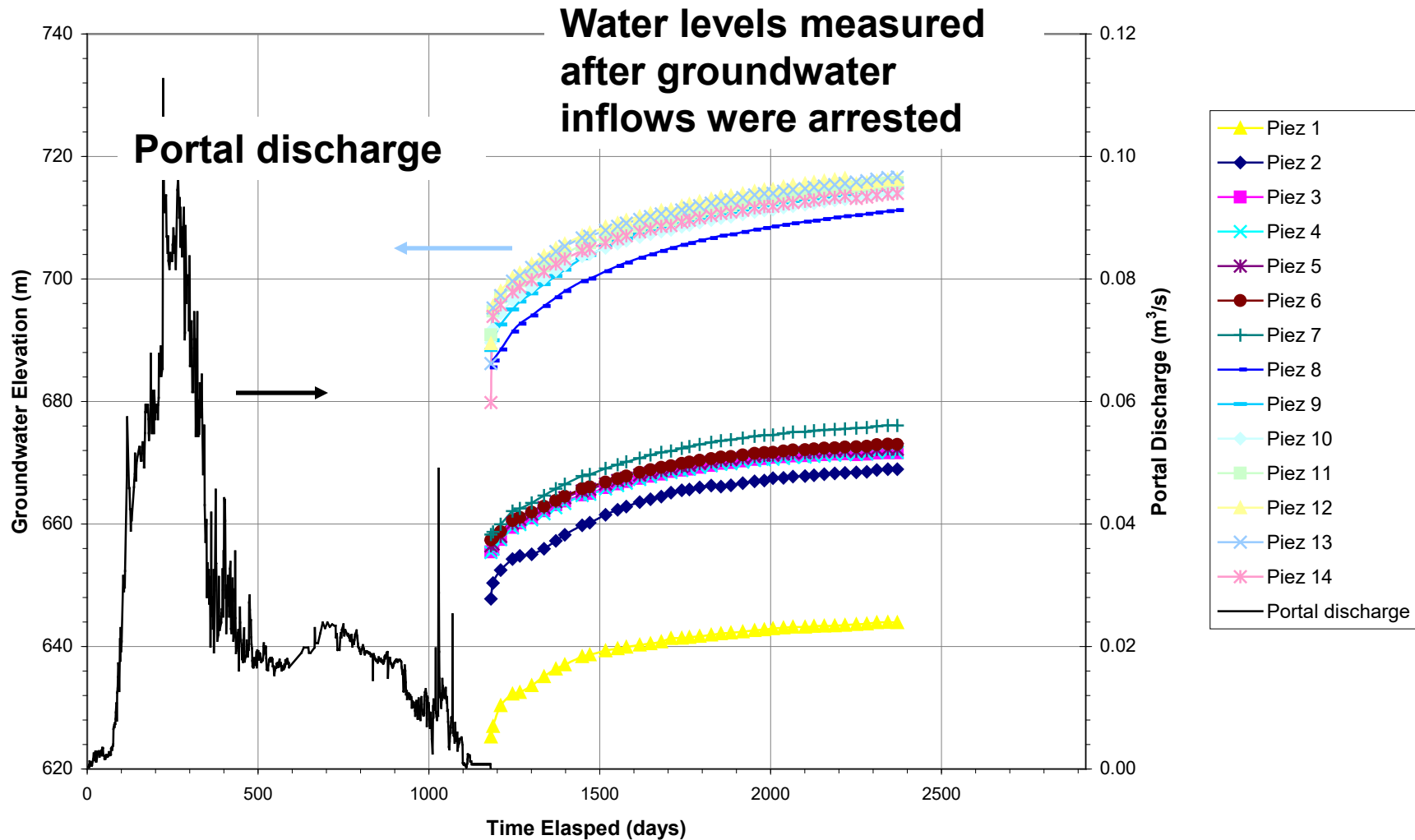
Recovery analyses may provide a useful check on the interpretations of the drawdown analyses.



The Cooper-Jacob recovery analysis is also useful for estimating pre-pumping conditions.

Case study: **Construction of the** **Arrowhead East Tunnel** **San Bernardino, California**





What were water levels prior to the start of tunnel construction?

Cooper-Jacob approximation

$$\begin{aligned}s(r, t) &= h_0 - h(r, t) \\ &= 2.303 \frac{Q}{4\pi T} \log_{10} \left\{ \frac{t}{t'} \right\}\end{aligned}$$

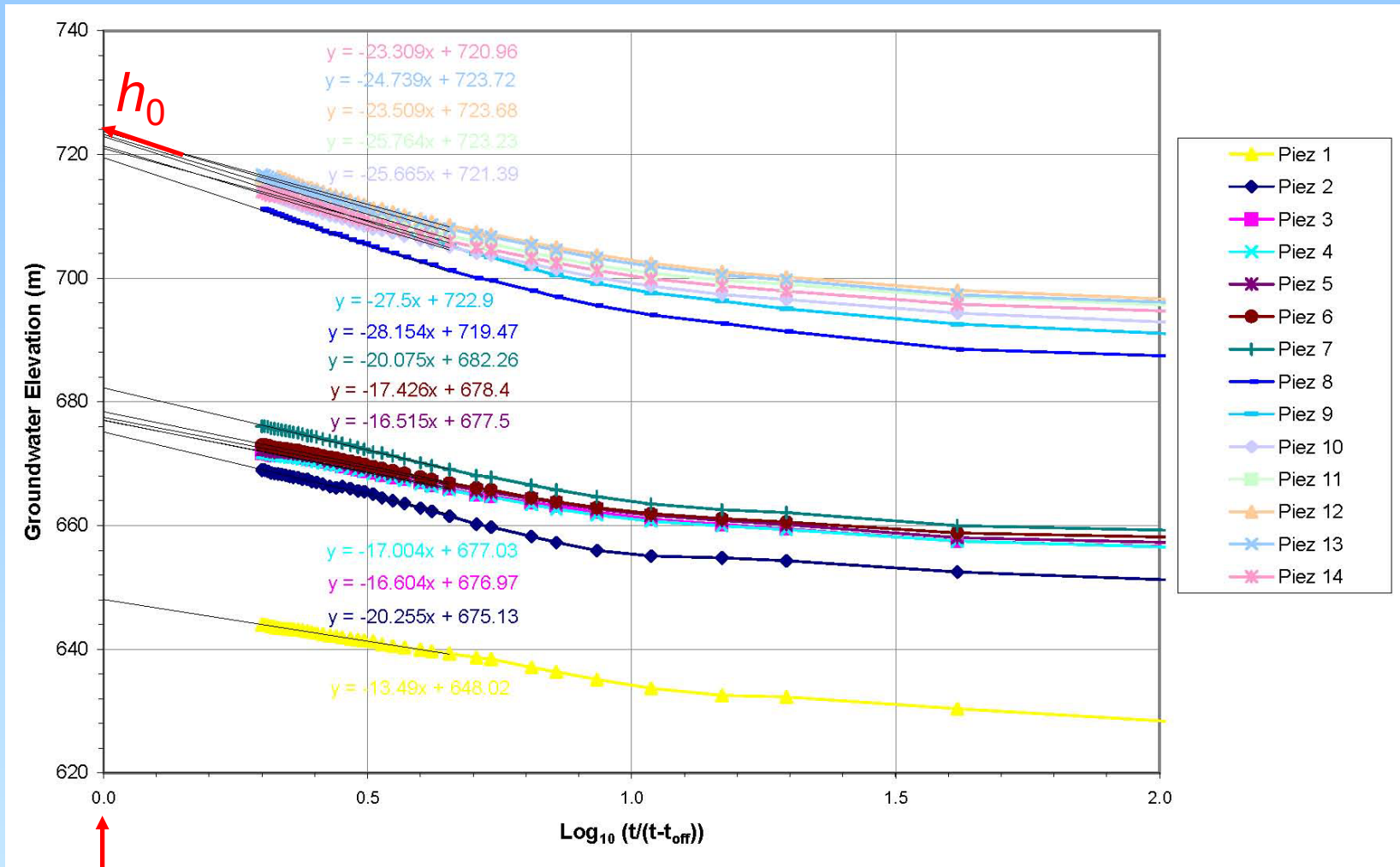
Re-arranging to solve for h_0 :

$$h_0 = h(r, t) - \frac{Q}{4\pi T} 2.303 \log_{10} \left\{ \frac{t}{t'} \right\}$$

As $\frac{t}{t'} \rightarrow 1$, $h(r, t) \rightarrow h_0$

aka
Horner analysis

Estimation of water levels prior to the start of tunnel construction



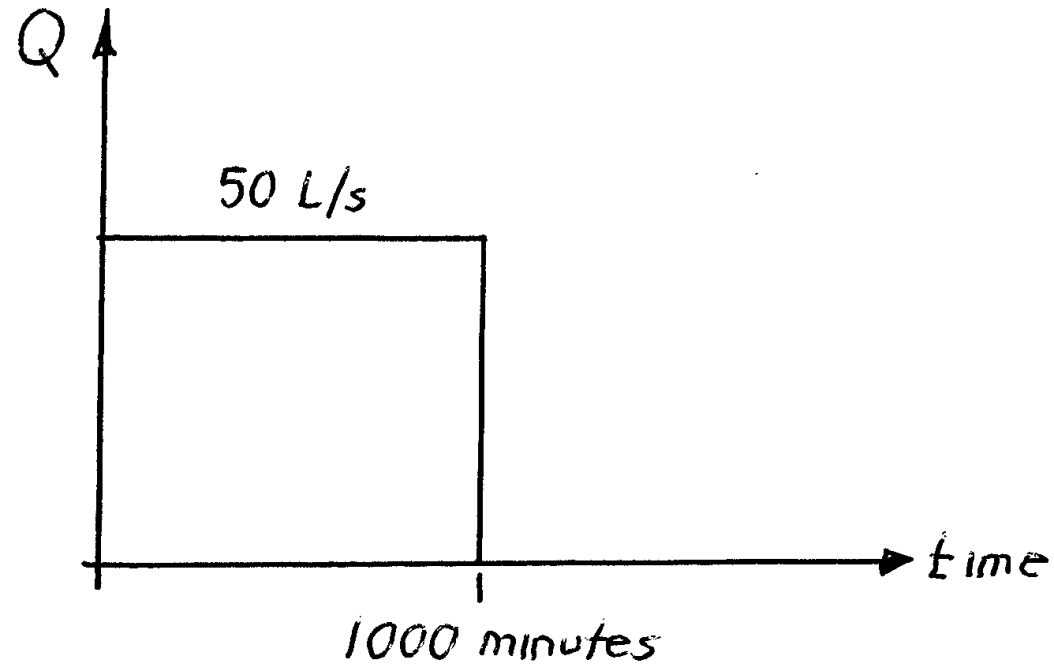
$t/t' \rightarrow 1$ (full recovery)

Insights from recovery data

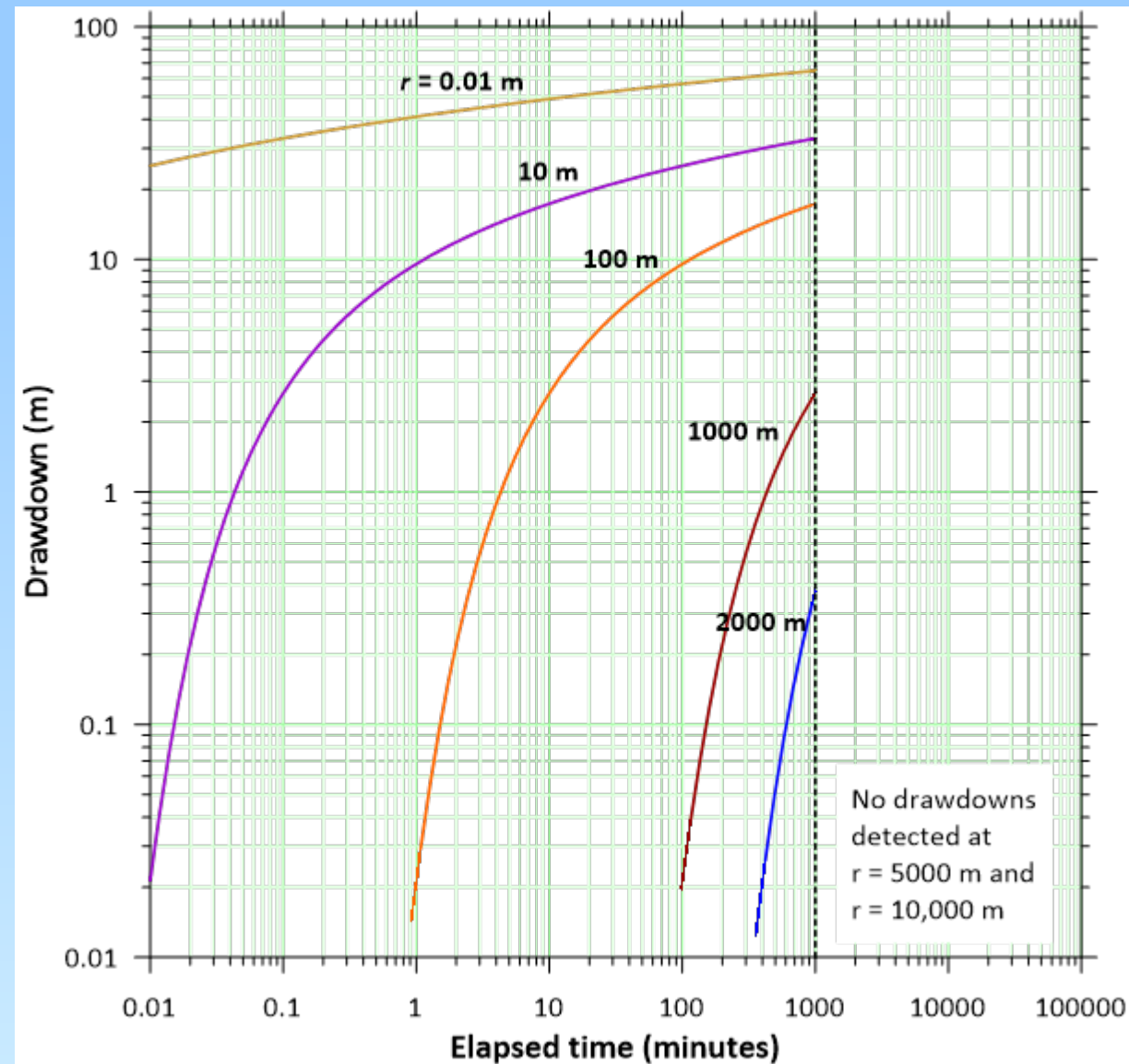
Recovery data may tell us more about our aquifer than we can glean from the drawdown data alone.

Example analysis

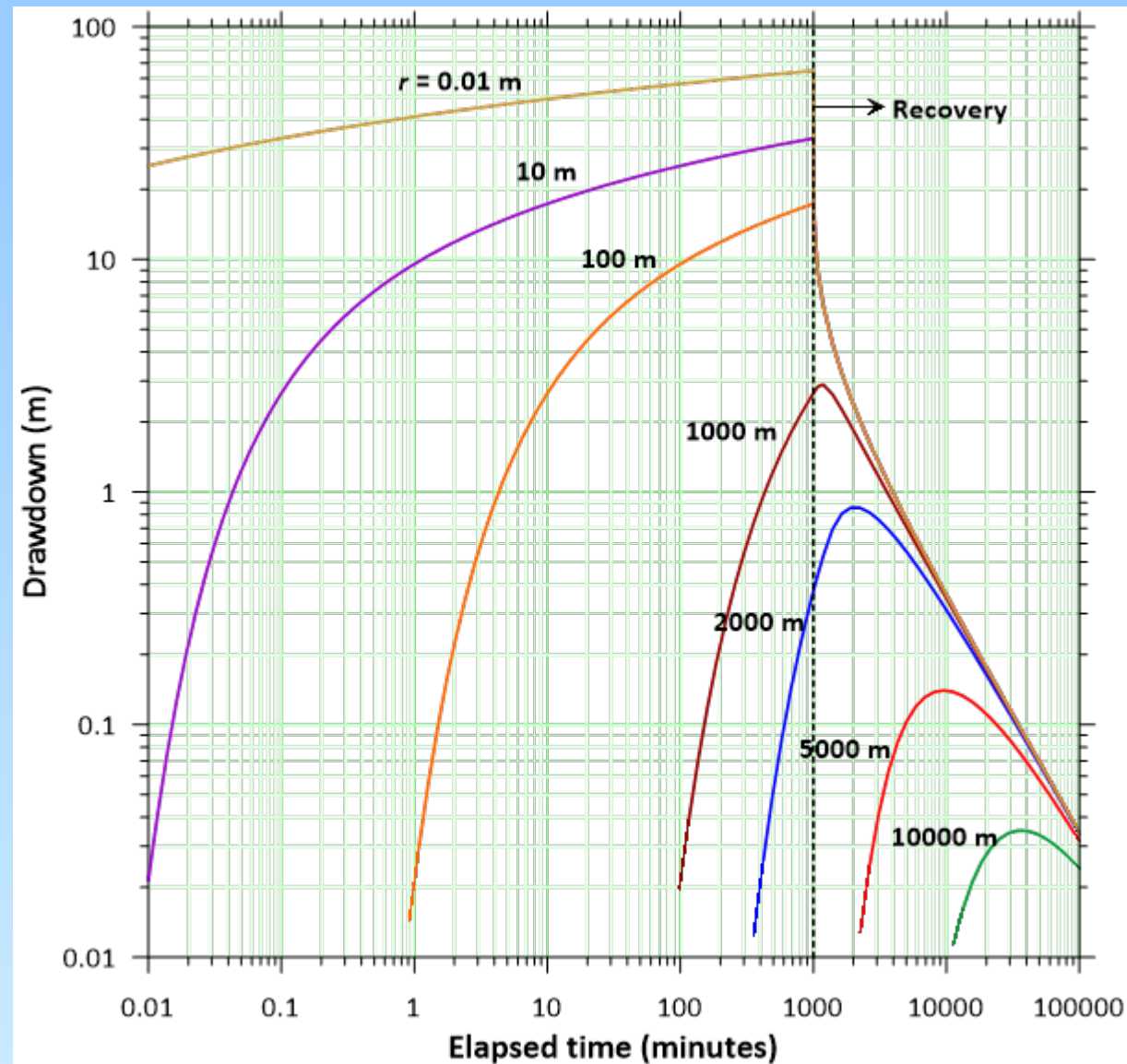
$$T = 100 \text{ m}^2/\text{d}$$
$$S = 1.0 \times 10^{-4}$$



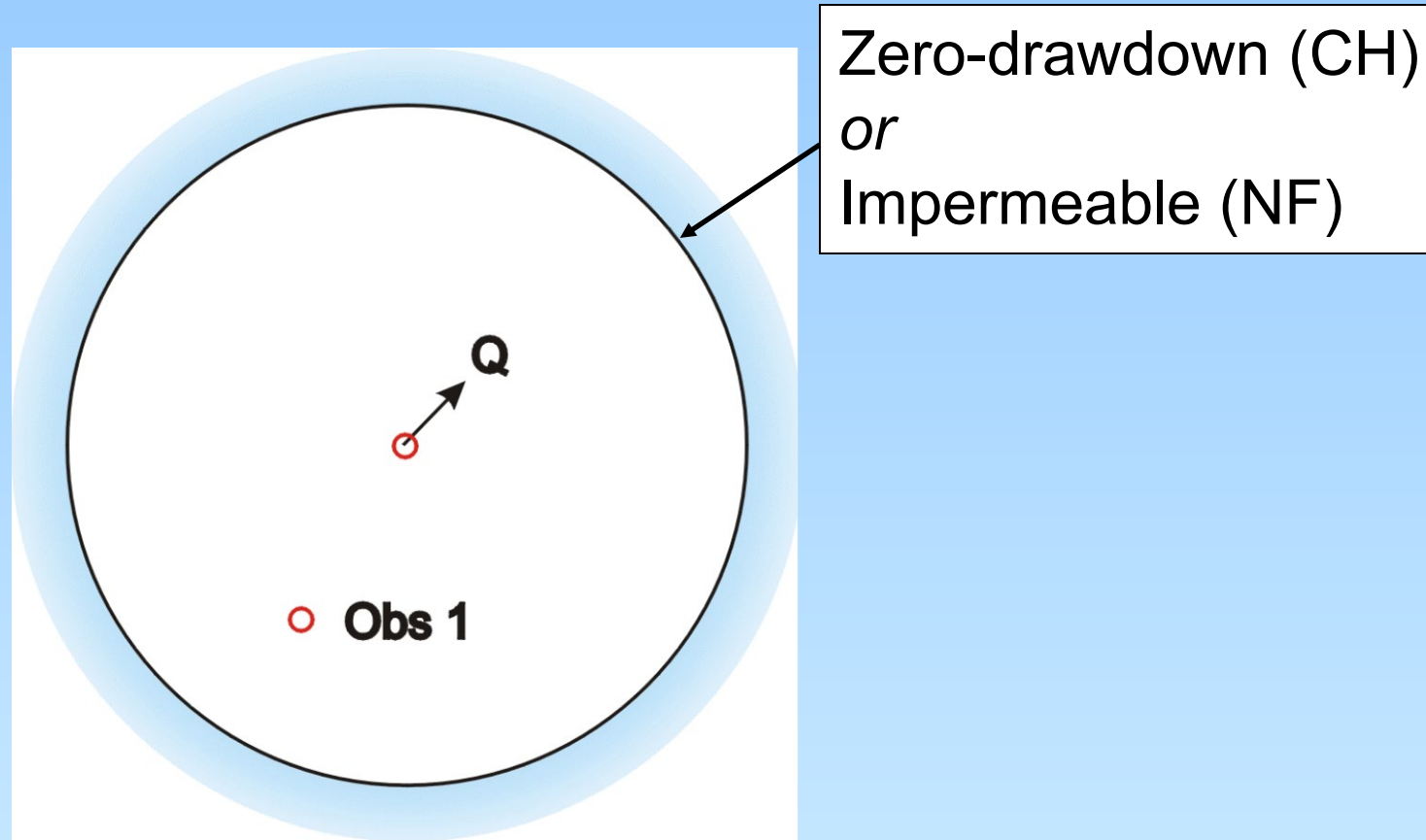
Drawdown records during the pumping



Complete drawdown records



Example:
Pumping in a finite circular aquifer

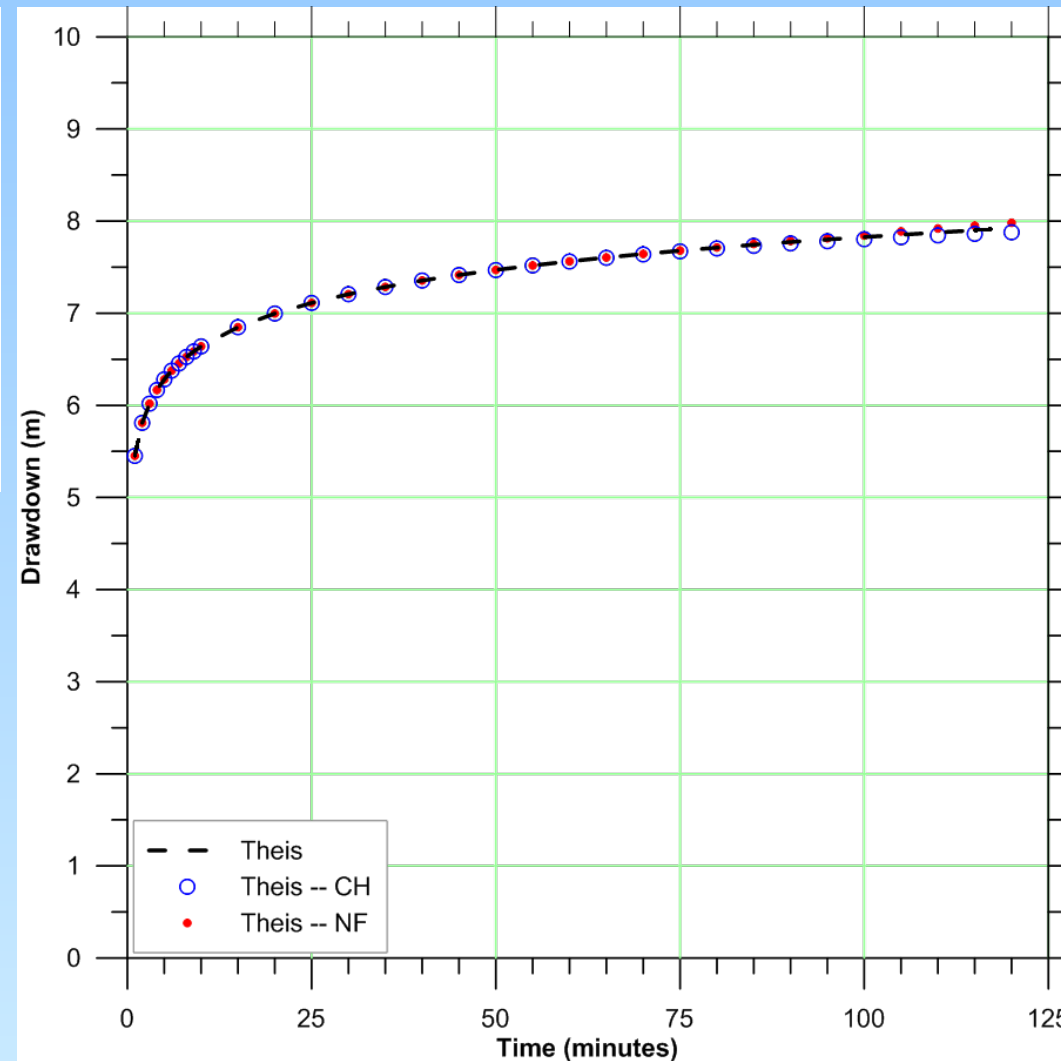


Pumping for 120 minutes: Drawdown record

Theis: Infinite aquifer

Theis-CH: Zero-drawdown
outer boundary

Theis-NF: Impermeable
outer boundary

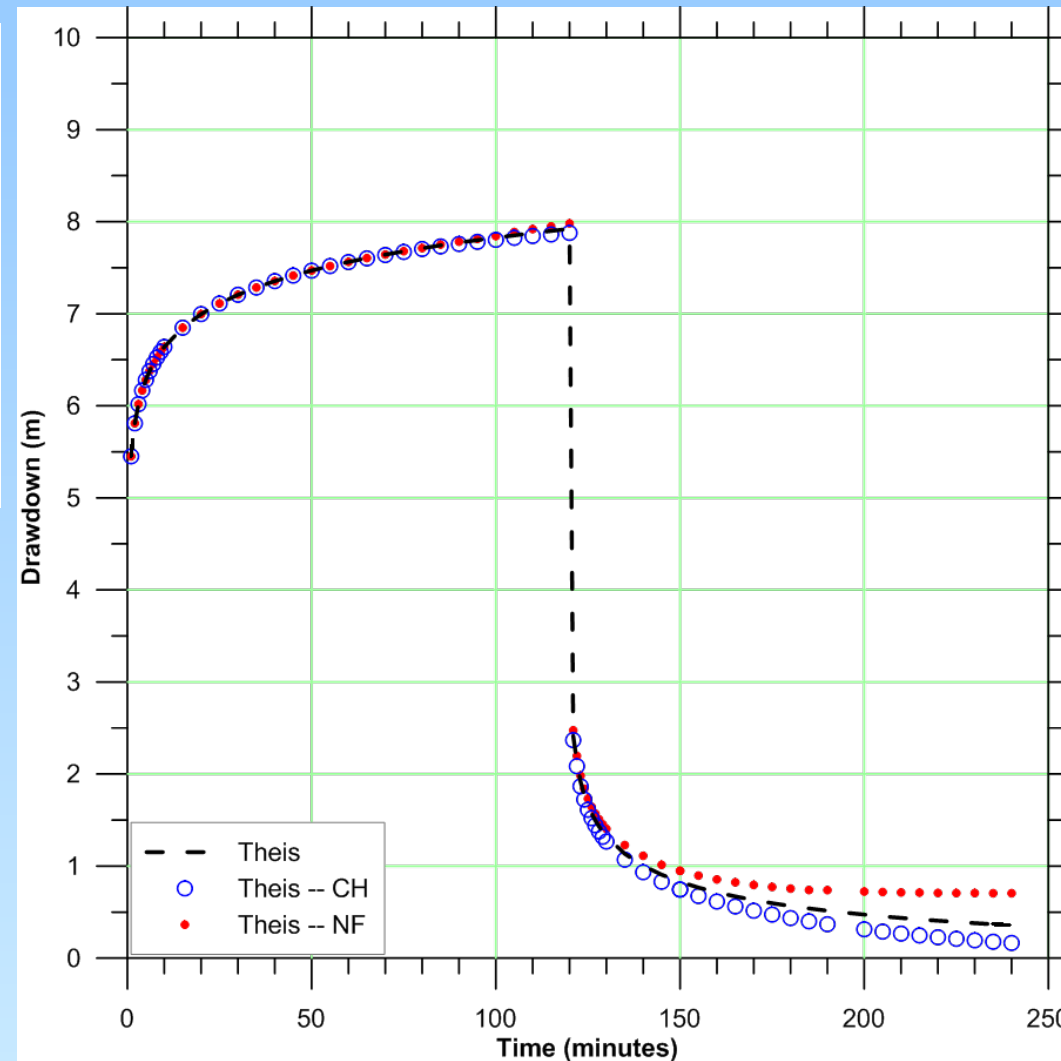


Complete drawdown and recovery record

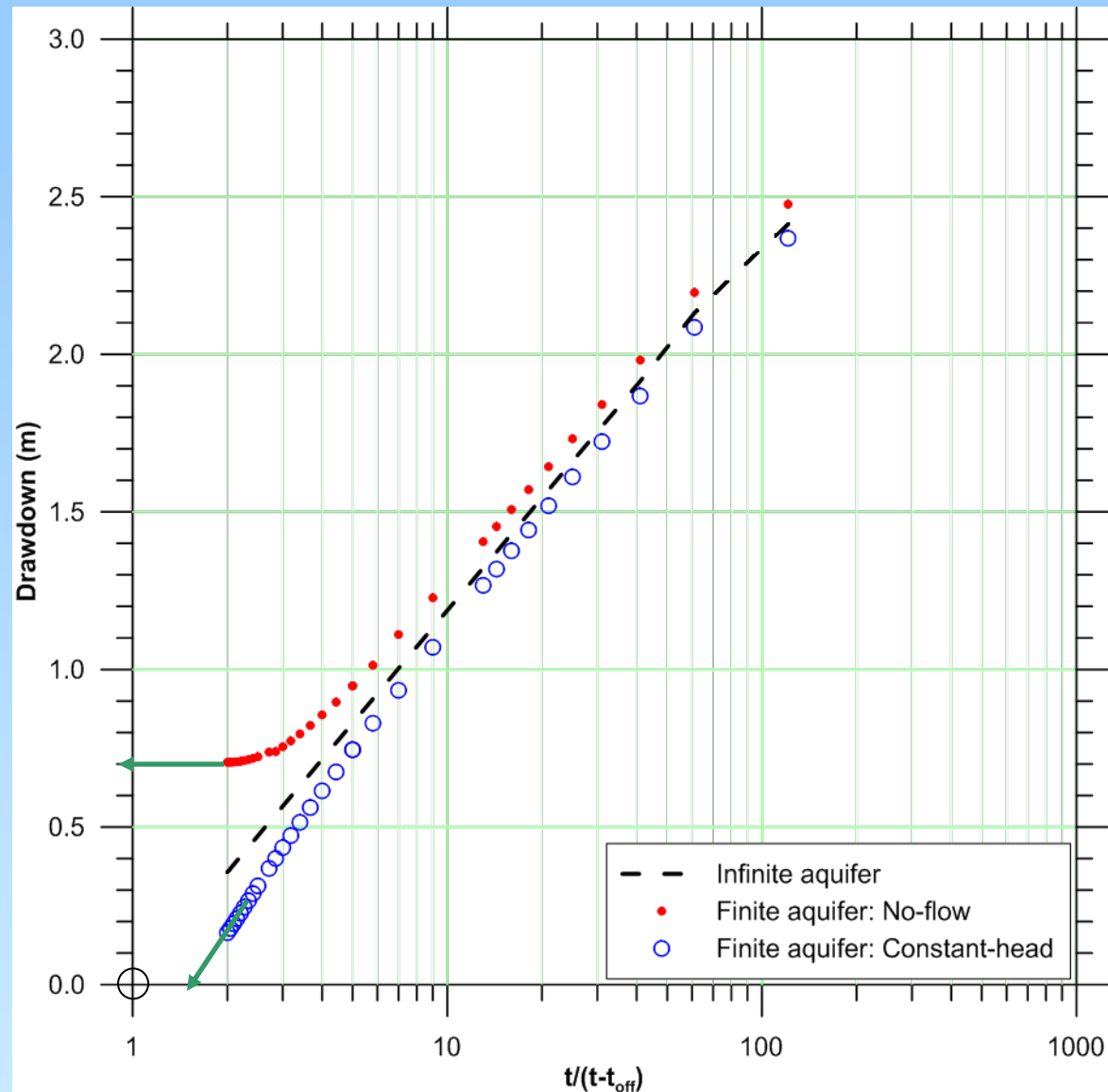
Theis: Infinite aquifer

Theis-CH: Zero-drawdown
outer boundary

Theis-NF: Impermeable
outer boundary



Cooper-Jacob recovery analysis



Use of recovery data to extend the effective duration of pumping

Calculation of Constant-Rate Drawdowns from Stepped-Rate Pumping Tests

by G. van der Kamp²

Vol. 27, No. 2—GROUND WATER—March-April 1989

Equivalent drawdown

The drawdown $s_1(r,t)$ that would have been observed if the well had remained pumping continuously at its initial rate Q_1 .

General formula for a linear model

$$s(r, t) = \int_0^t Q(\tau) G(r, t - \tau) d\tau$$

“Impulse function”

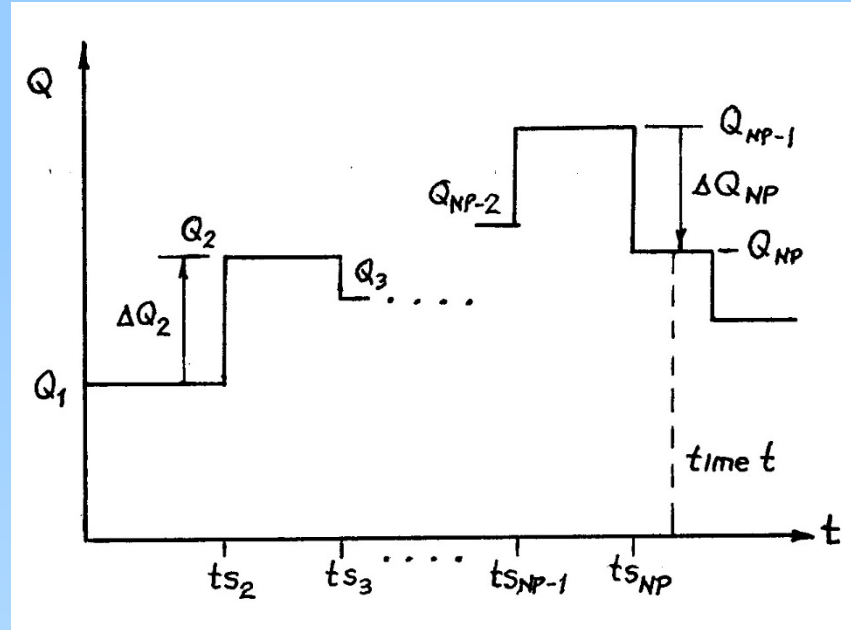
Actual pumping history

Equivalent drawdown:

$$s_1(r, t) = Q_1 \int_0^t G(\tau) d\tau$$

General formula for van der Kamp's method

For a pumping history represented by a set of steps



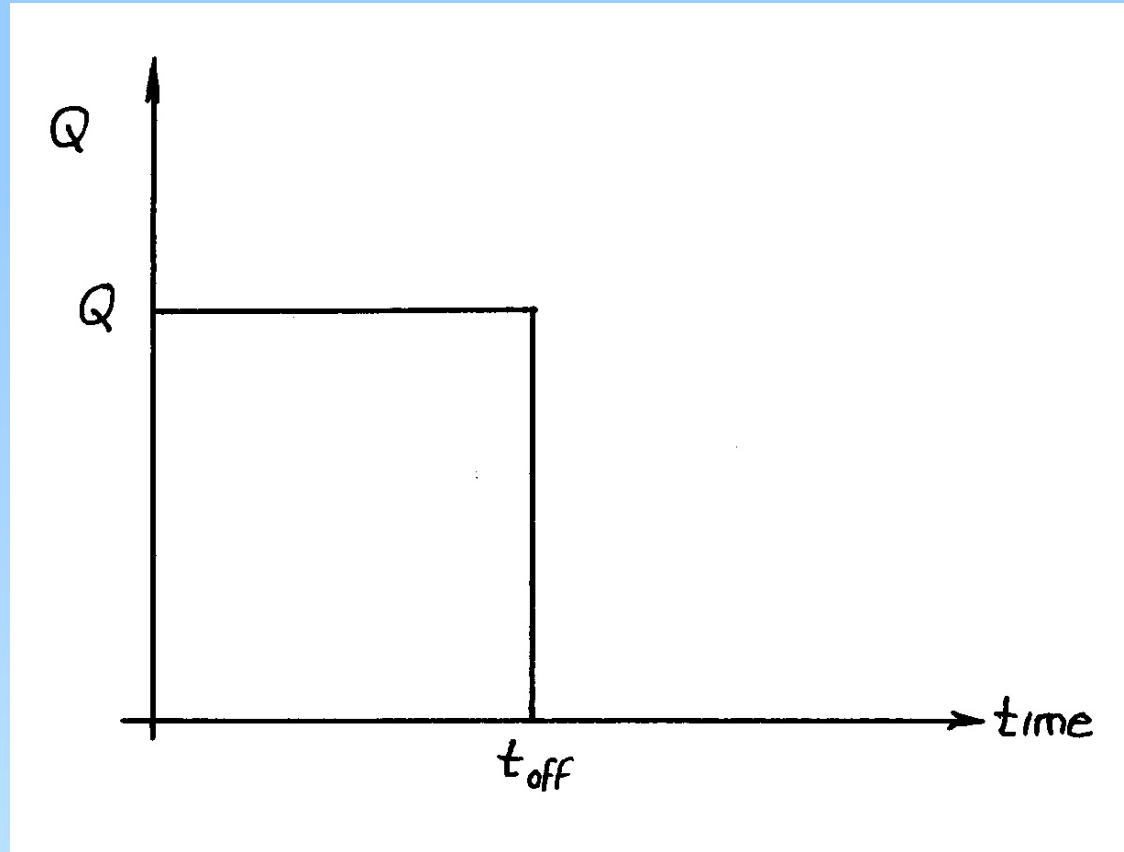
$$s_1(r, t) = \boxed{s(r, t)} \leftarrow \text{Observed drawdowns}$$

$$- \frac{(Q_2 - Q_1)}{Q_1} s_1(r, t - ts_2)$$

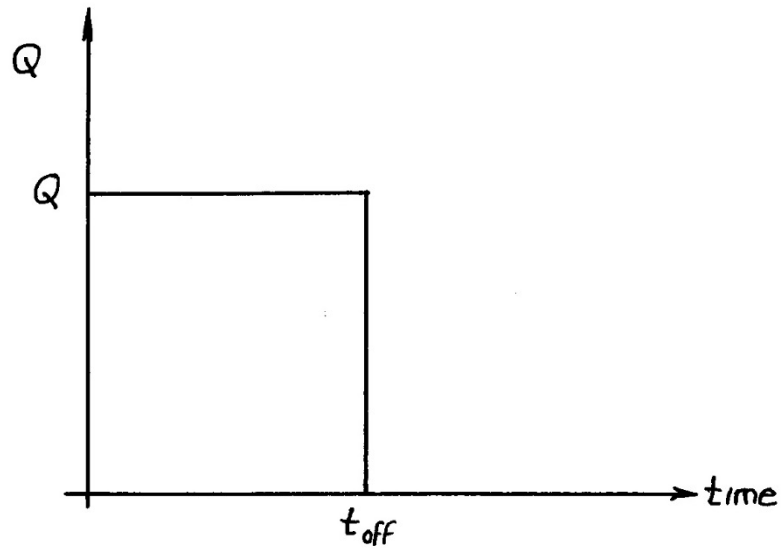
$$\dots$$

$$- \frac{(Q_{NP} - Q_{NP-1})}{Q_1} s_1(r, t - ts_{NP})$$

Pumping at a constant rate followed by recovery



In this context, the *equivalent drawdown* is that drawdown that we would have observed if the well had continued pumping.

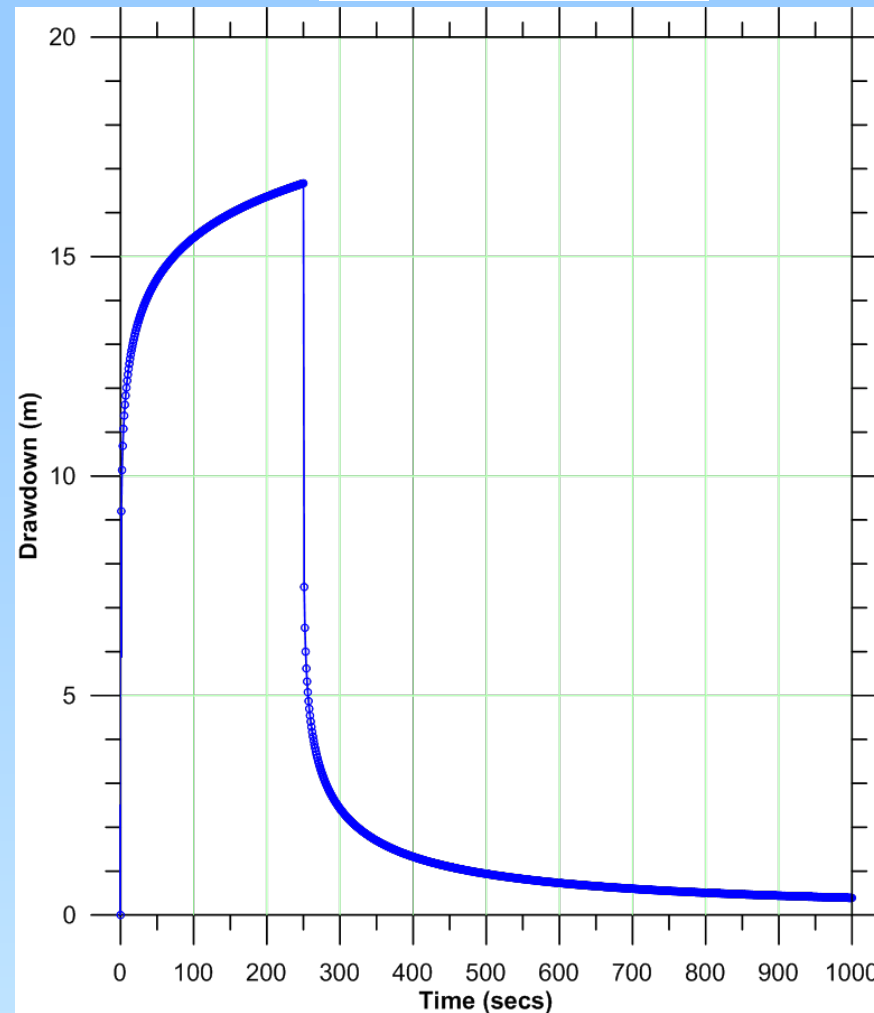


- $NP = 2$
- $ts2 = t_{off}$; $Q_2 = 0.0$

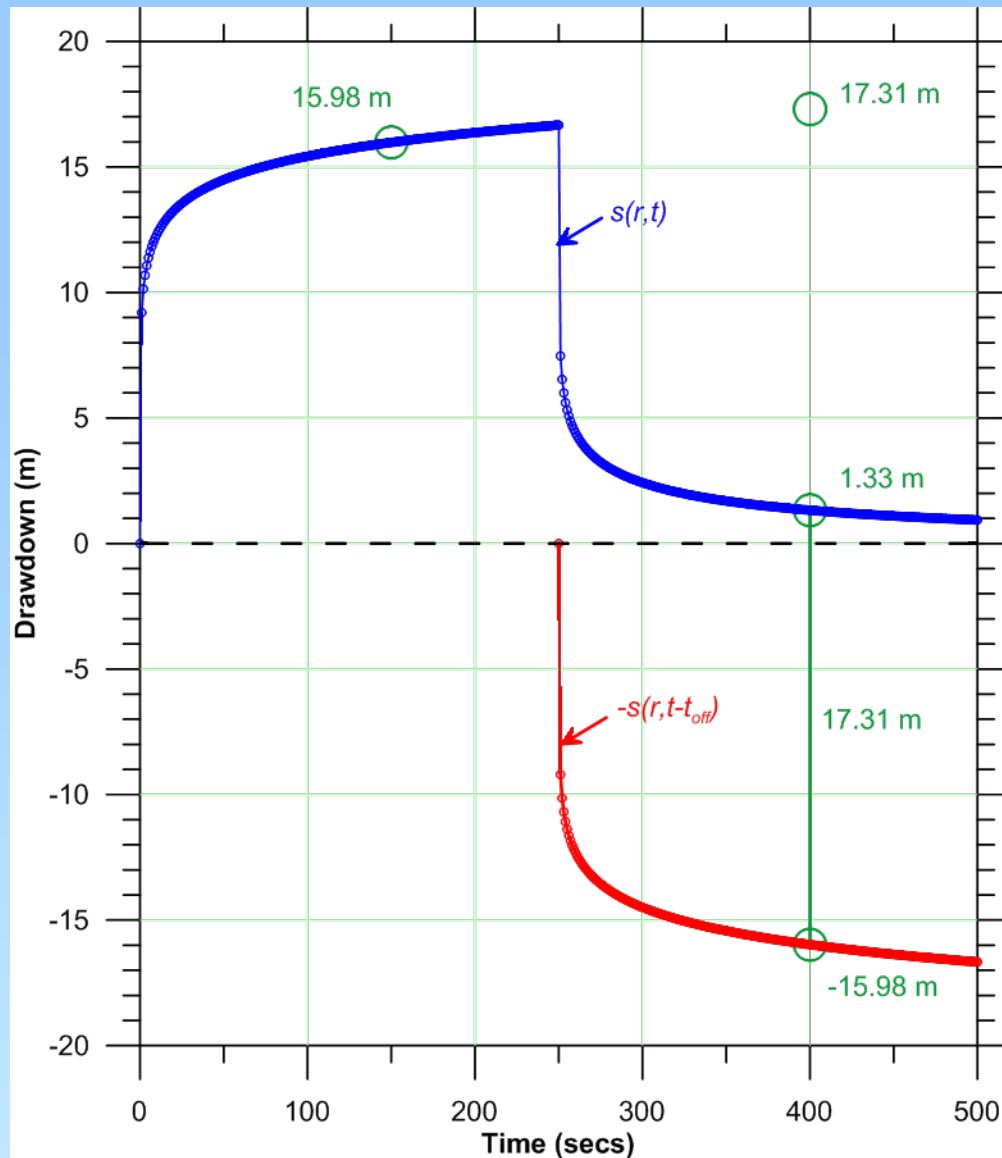
$$s_1(r, t) = s(r, t) - \frac{([0] - Q_1)}{Q_1} s_1(r, t - [t_{off}])$$

$$\rightarrow s_1(r, t) = s(r, t) + s_1(r, t - t_{off})$$

Example



Pumping stopped after 250 seconds.
What drawdown would we have observed at 400 seconds if the well had continued pumping?

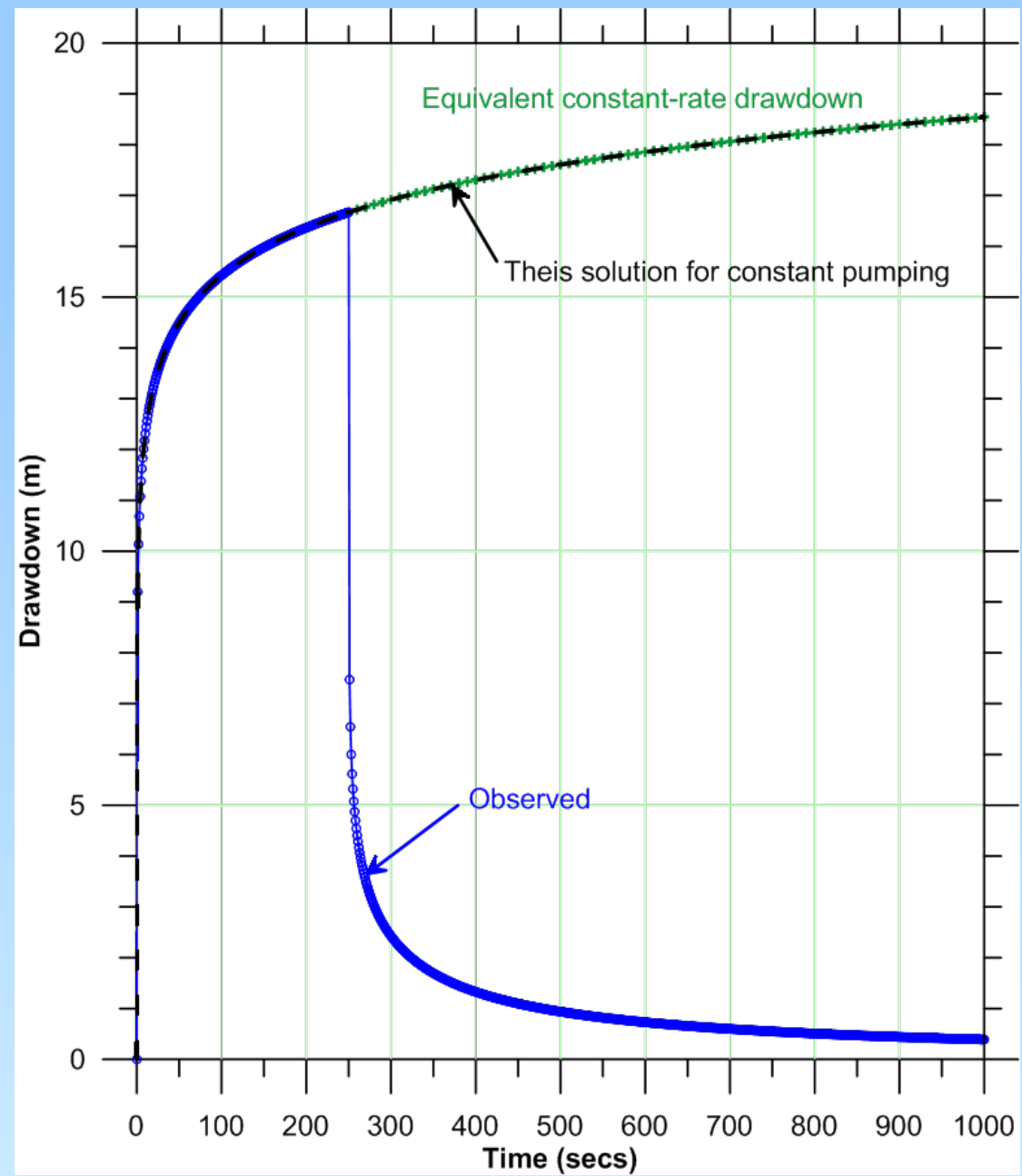


$$s_1(r, t) = s(r, t) + s_1(r, t - t_{off})$$

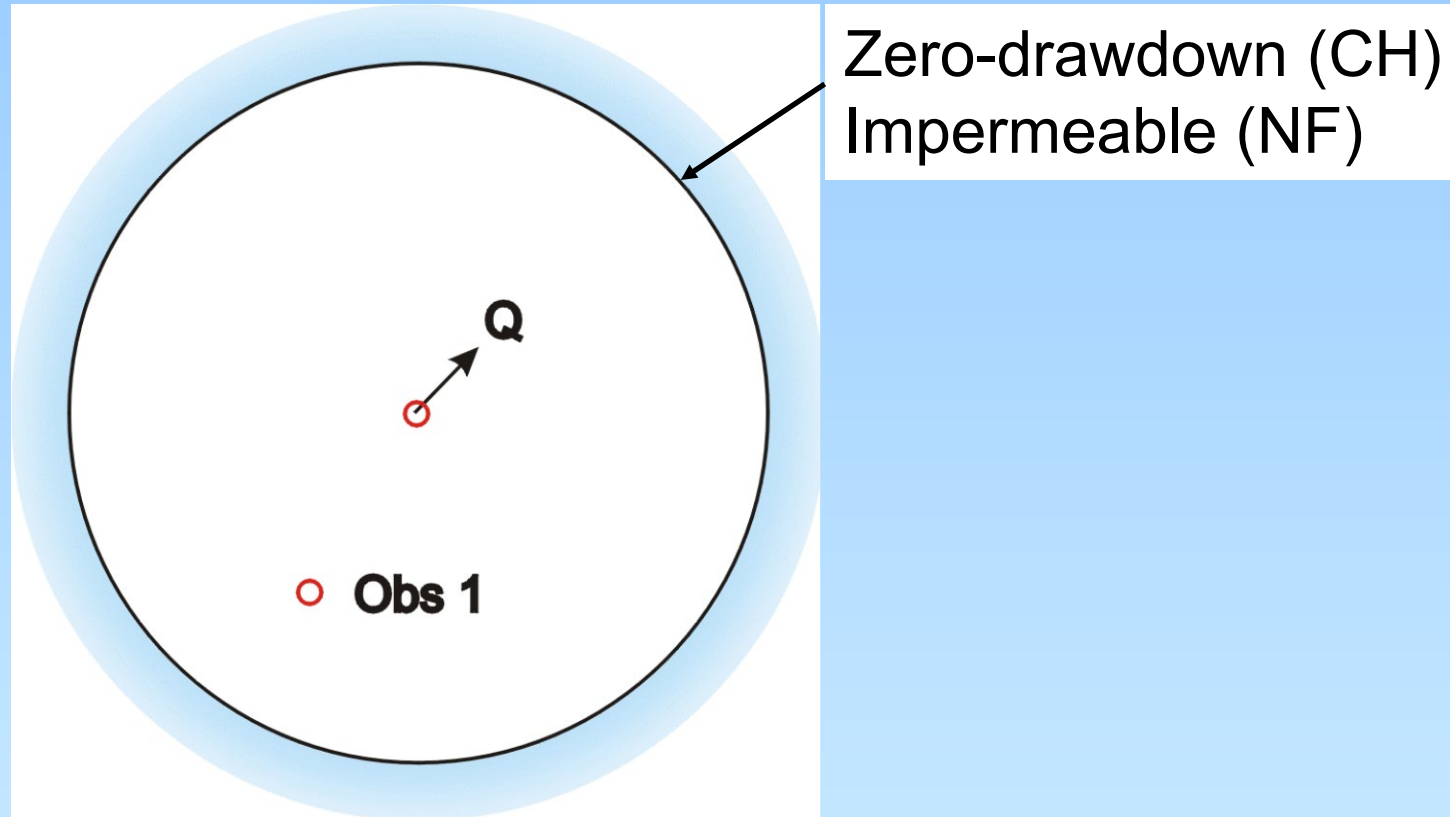
@ $t = 400$ s
 $s(400 \text{ s}) = 1.33 \text{ m}$

$t - t_{off} = 400 - 250 = 150 \text{ s}$
 $s(150 \text{ s}) = 15.98 \text{ m}$

$$s_1(400 \text{ s}) = 1.33 \text{ m} + 15.98 \text{ m} = 17.31 \text{ m}$$

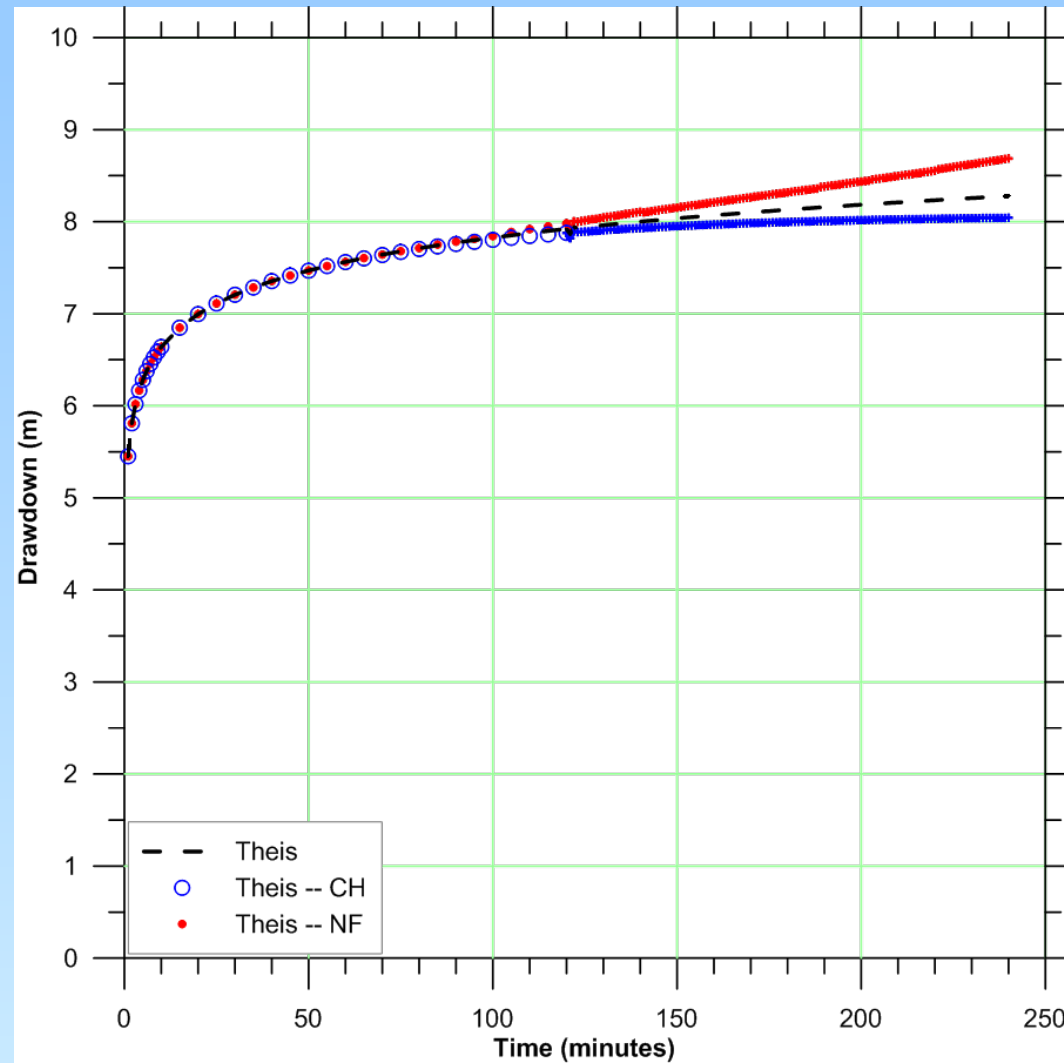


Pumping in a finite circular aquifer revisited

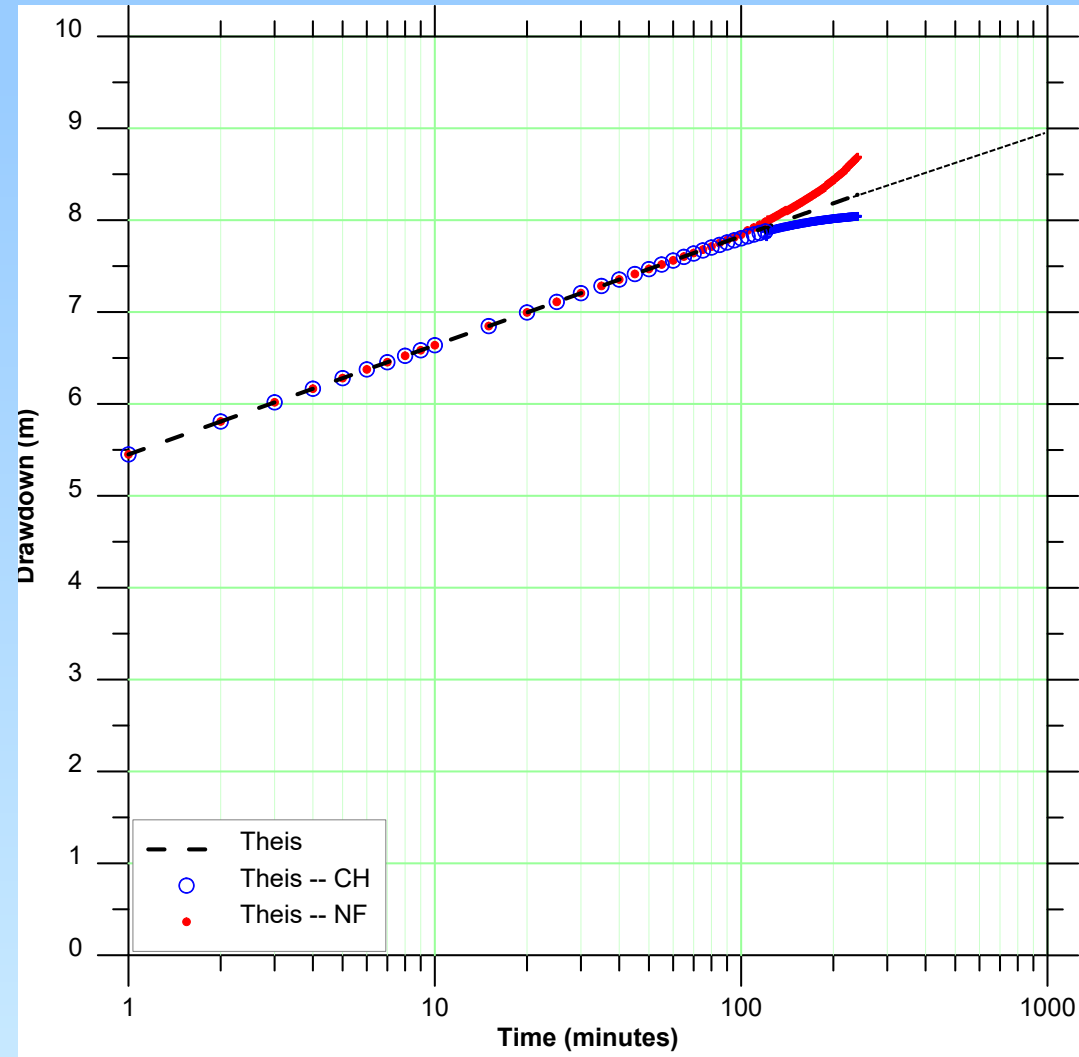


van der Kamp

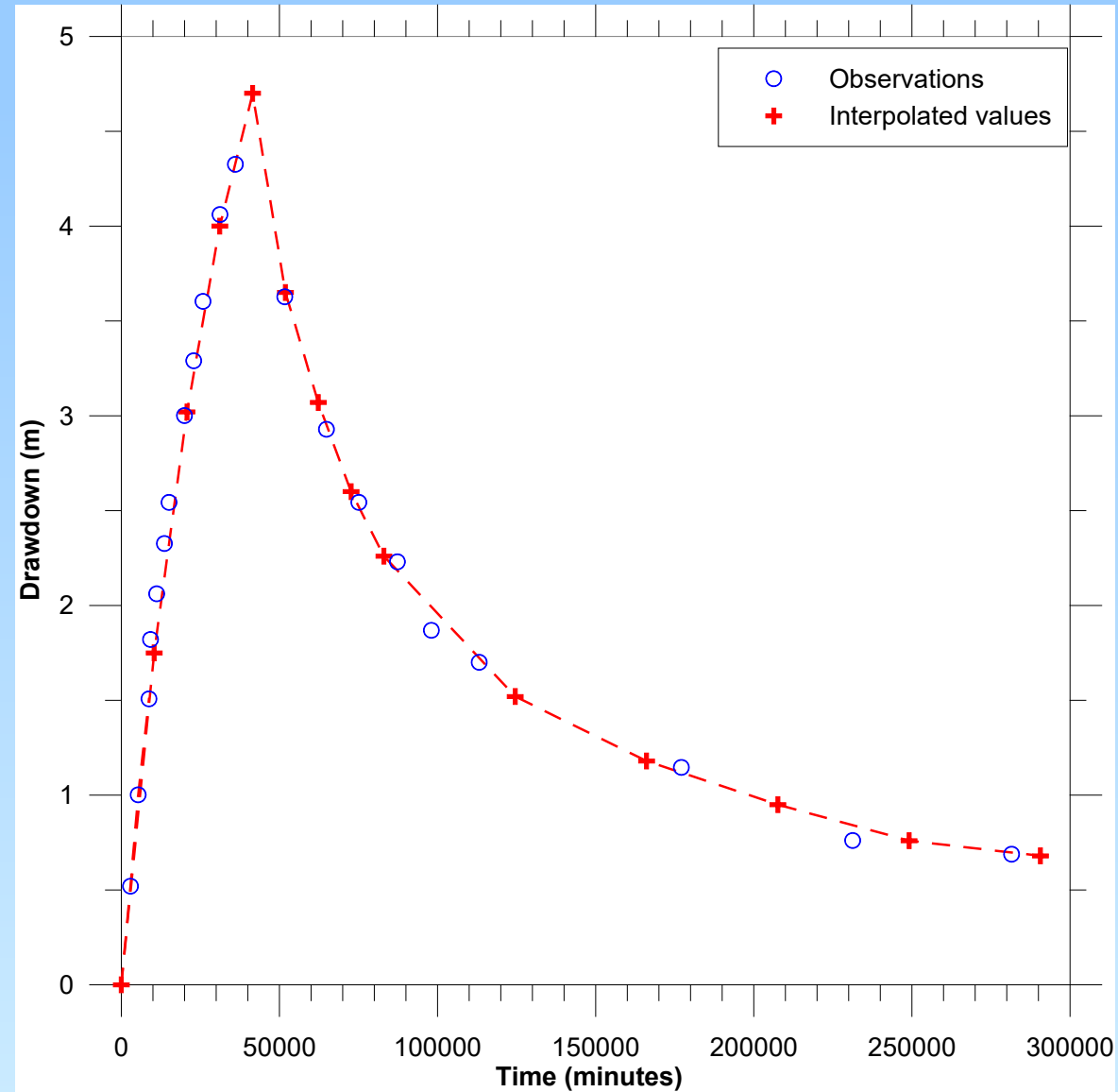
Equivalent constant-rate drawdowns



Drawdown record: Semilog plot with the extended drawdown record



Case study: Estevan, Saskatchewan



t	s (t)	t-t _{off}	s (t-t _{off})	s ₁
(minutes)	(m)	(minutes)	(m)	(m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07			
72660	2.60			
83040	2.26			
124560	1.52			
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

t	s (t)	t-t _{off}	s (t-t _{off})	s ₁
(minutes)	(m)	(minutes)	(m)	(m)
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20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60			
83040	2.26			
124560	1.52			
166080	1.18			
207600	0.95			
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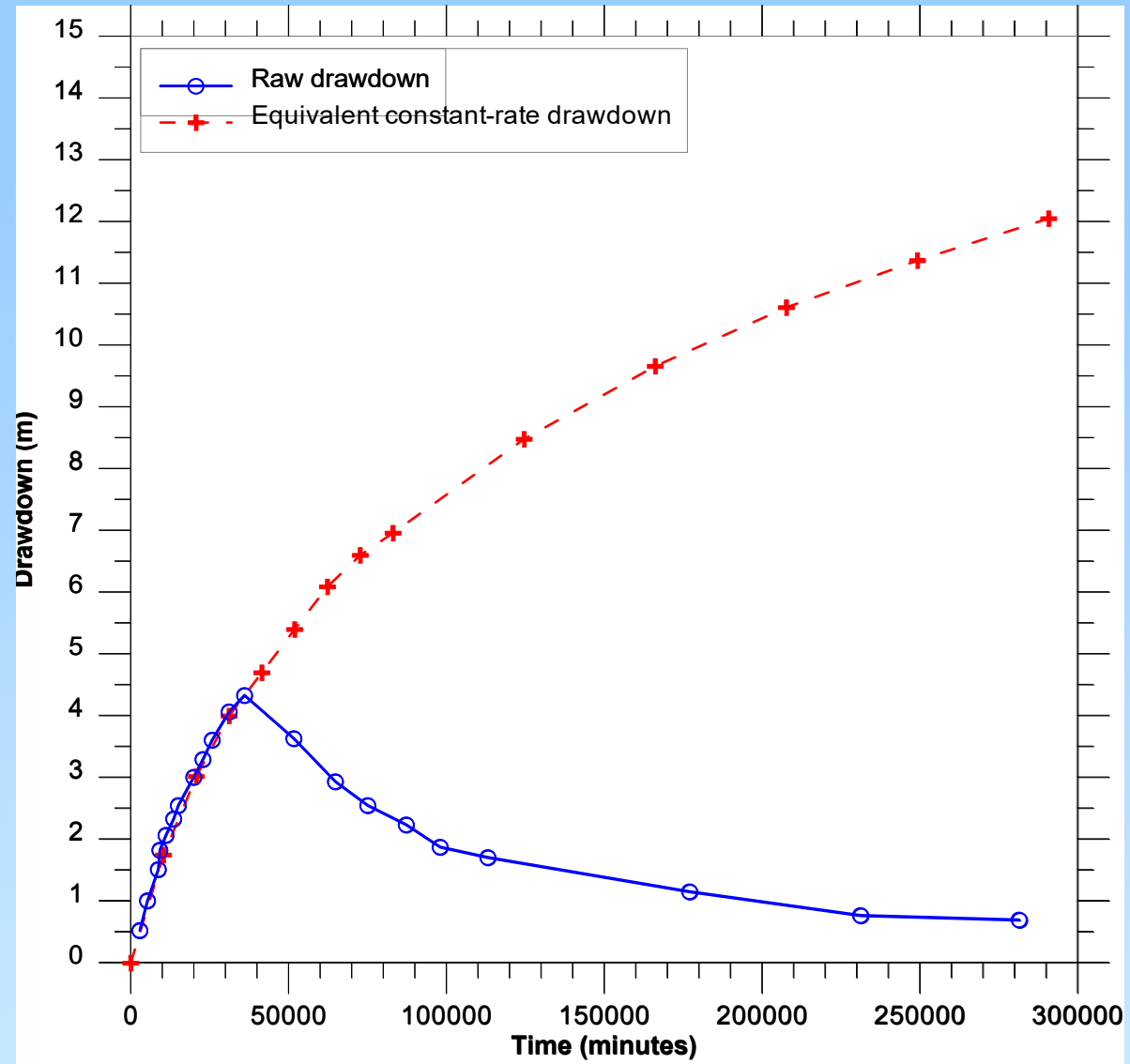
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72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

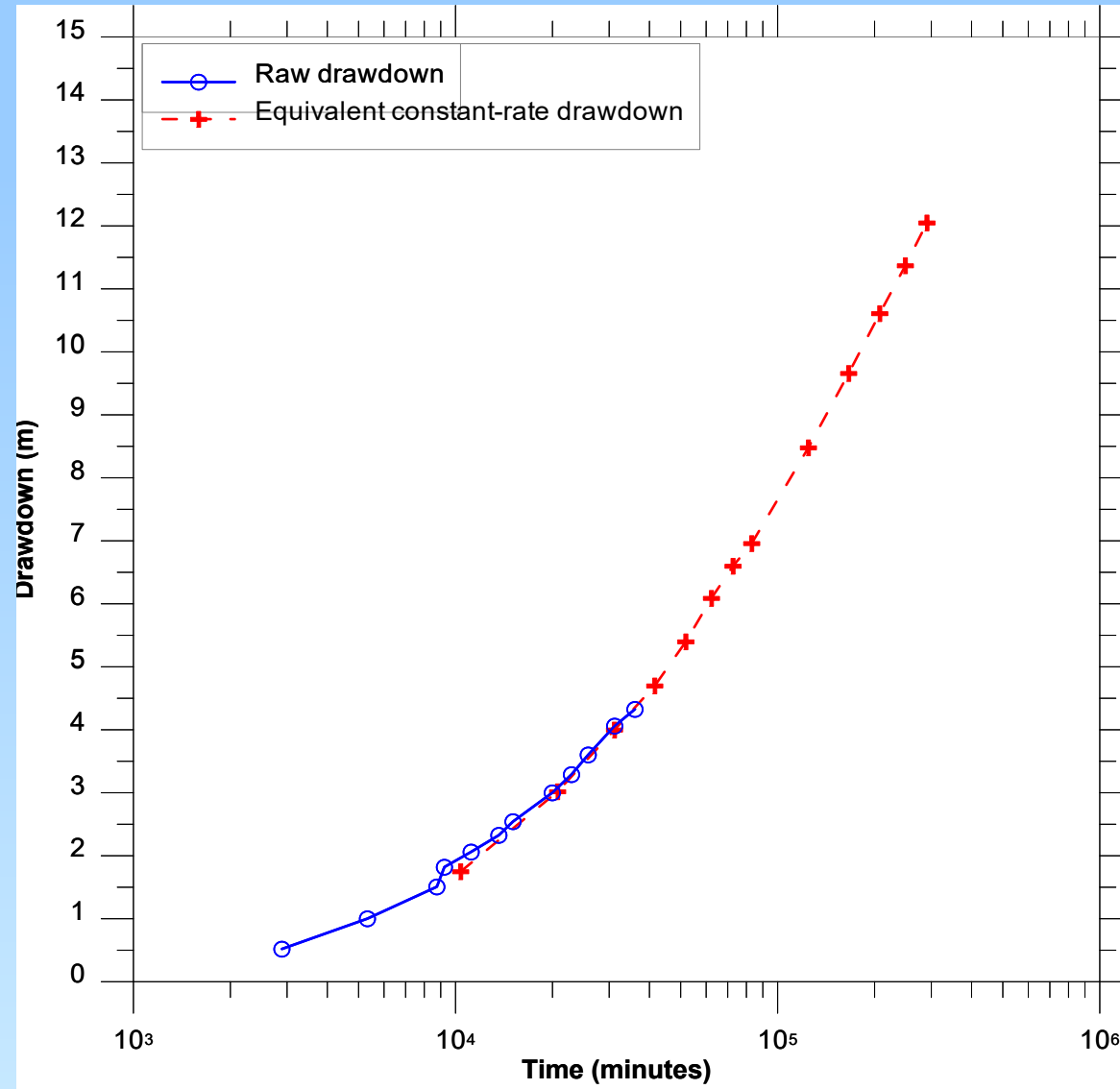
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124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95			
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10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76			
290640	0.68			

t	s (t)	t-t _{off}	s (t-t _{off})	s ₁
(minutes)	(m)	(minutes)	(m)	(m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76	207600	10.61	11.37
290640	0.68			

t	s (t)	t-t _{off}	s (t-t _{off})	s ₁
(minutes)	(m)	(minutes)	(m)	(m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76	207600	10.61	11.37
290640	0.68	249120	11.37	12.05

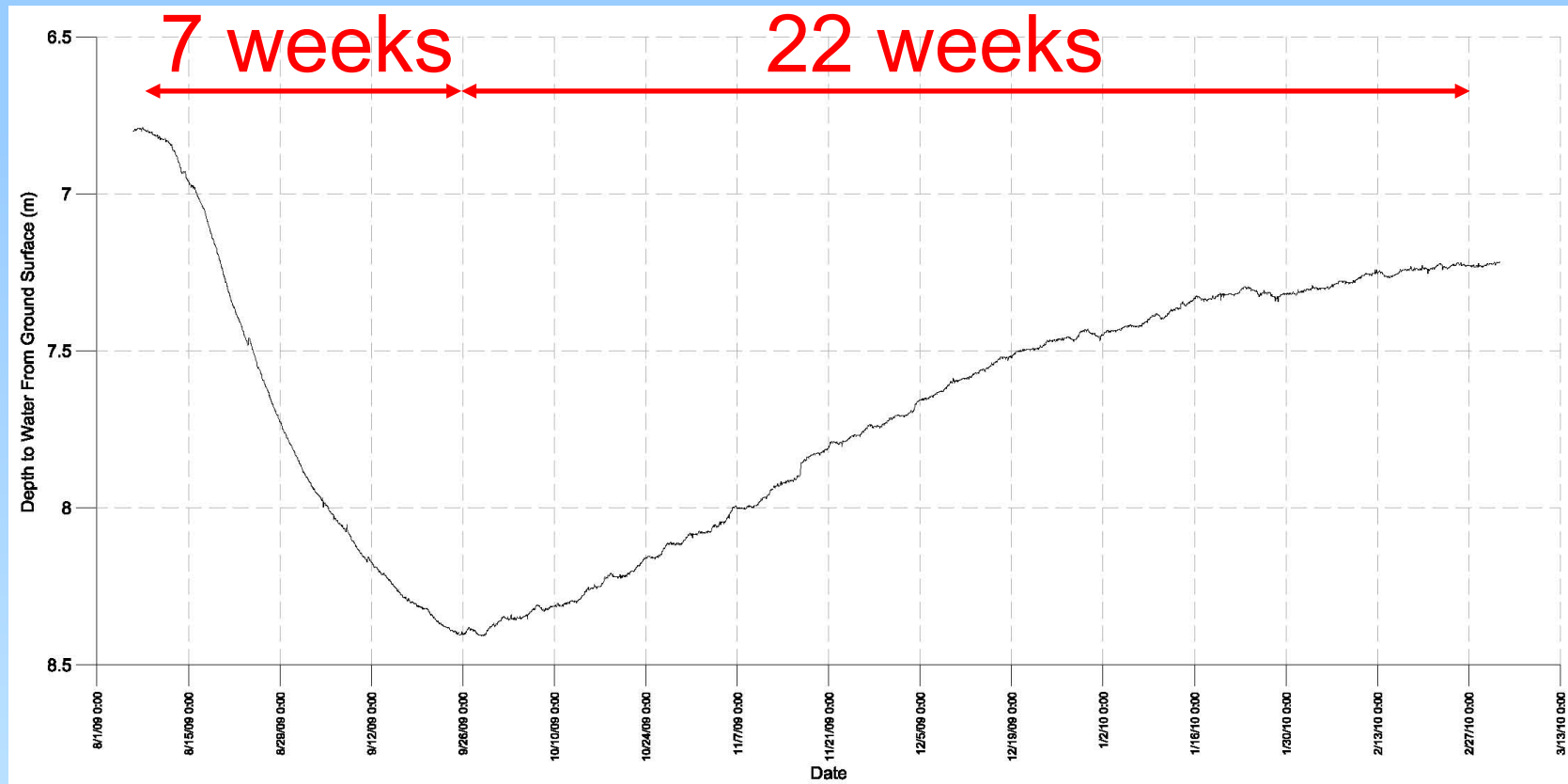




Take-home points

1. Recovery data may be some of the best data available.
2. Recovery data are straightforward to interpret using superposition.
3. Recovery data may offer insights into aquifer behaviour not available from drawdown data.
4. The van der Kamp method to use of recovery data to extend the effective duration of pumping is simple and useful.

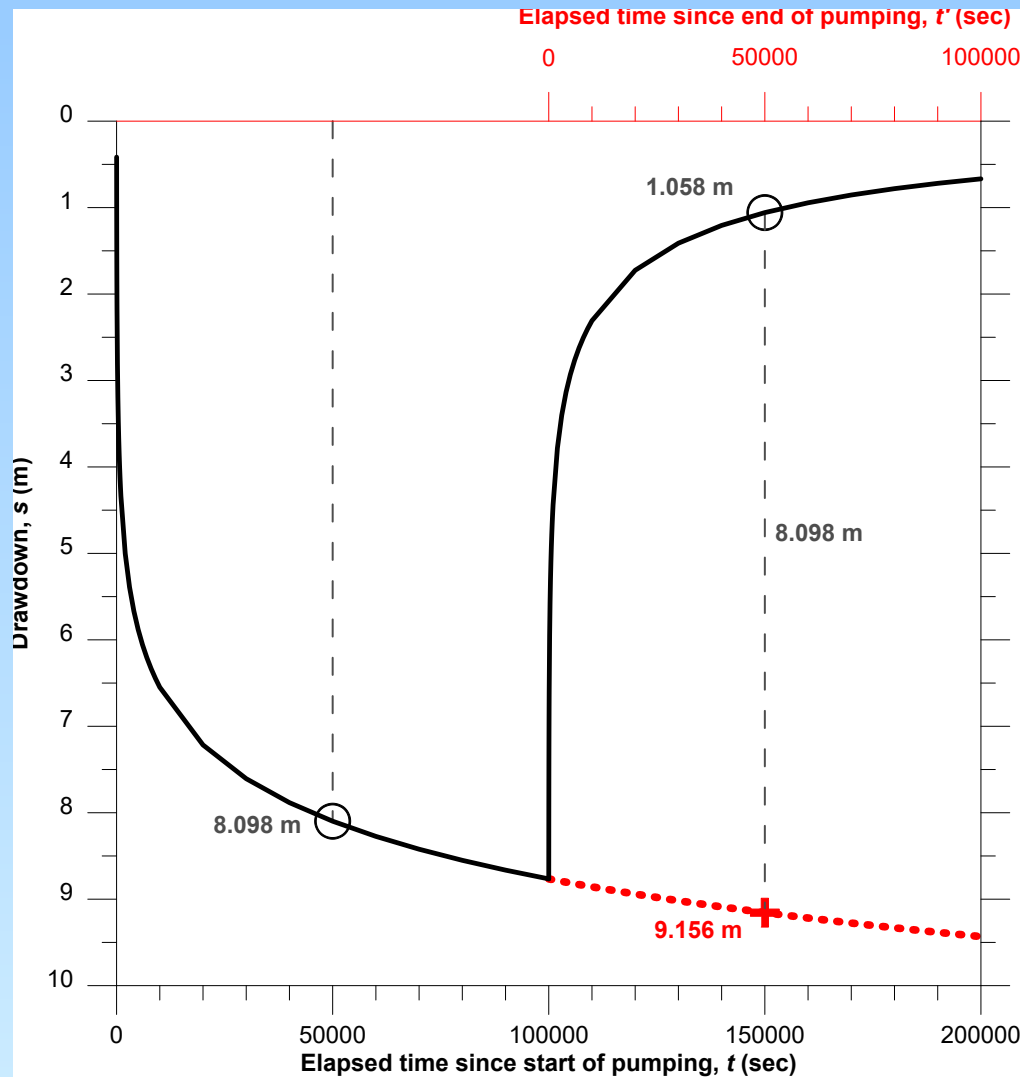
MW-1 observation well



Q: What drawdowns would have been observed if pumping had continued at a constant rate?

The van der Kamp (1989) method in a nutshell

$$s_1(r, t) = s(r, t) + s_1(r, t - t_{off})$$



What would the drawdown after 15,000 seconds have been if pumping had continued?

$$s_1(t = 15,000 \text{ s}) \\ = s(t = 15,000 \text{ s}) \\ + s_1(t = 15,000 \text{ s} - 10,000 \text{ s})$$

$$= 1.058 \text{ m} + 8.098 \text{ m} = \mathbf{9.156 \text{ m}}$$

